Cambridge Assessment
International Education

Cambridge
Pre-U



# SYLLABUS

## Cambridge International Level 3 Pre-U Certificate in Mathematics (Principal)

## 9794

## For centres in the UK

For examination in 2022

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate. QN: 500/3789/4

Cambridge Advanced

Version 1

**Changes to the syllabus for 2022**

The latest syllabus is version 1, published September 2019.

There are no significant changes which affect teaching.

**You are strongly advised to read the whole syllabus before planning your teaching programme.**

# Contents

# Introduction

## Why choose Cambridge Pre-U?

Cambridge Pre-U is designed to equip learners with the skills required to make a success of their studies at university. Schools can choose from a wide range of subjects.

Cambridge Pre-U is built on a core set of educational aims to prepare learners for university admission, and also for success in higher education and beyond:

- to support independent and self-directed learning
- to encourage learners to think laterally, critically and creatively, and to acquire good problem-solving skills
- to promote comprehensive understanding of the subject through depth and rigour.

Cambridge Pre-U Principal Subjects are linear. A candidate must take all the components together at the end of the course in one examination series. Cambridge Pre-U Principal Subjects are assessed at the end of a two-year programme of study.

The Cambridge Pre-U nine-point grade set recognises the full range of learner ability.

## Why choose Cambridge Pre-U Mathematics?

- Cambridge Pre-U Mathematics is designed to encourage teaching and learning which enable learners to develop a positive attitude towards the subject by developing an understanding of mathematics and mathematical processes in a way that promotes confidence and enjoyment.
- Throughout this course, learners are expected to develop two parallel strands of mathematics: pure mathematics and applications of mathematics.
- The study of mathematics encourages the development of logical thought and problem-solving skills.
- The linear assessment structure means that learners are tested at the end of the two-year course. This allows learners to approach the examination in a mature and confident way, with time to develop their knowledge, understanding and skills.
- Cambridge Pre-U Mathematics involves the acquisition of skills that can be applied in a wide range of contexts.

### Prior learning

Cambridge Pre-U builds on the knowledge, understanding and skills gained by learners achieving a good pass in Level 1/Level 2 qualifications in mathematics or related subjects.

## Progression

Cambridge Pre-U is considered to be an excellent preparation for university, employment and life. It helps to develop the in-depth subject knowledge and understanding which are so important to universities and employers. While it is a satisfying subject in its own right, mathematics is also a prerequisite for further study in an increasing range of subjects. For this reason, learners following this course will be expected to apply their mathematical knowledge in the contexts of both mechanics and probability, and will also be presented with less familiar scenarios. This syllabus provides a sound foundation in mathematics for higher education courses or other career pathways.

Cambridge Assessment International Education is an education organisation and politically neutral. The content of this syllabus, examination papers and associated materials do not endorse any political view. We endeavour to treat all aspects of the exam process neutrally.

## Cambridge Pre-U Diploma

If learners choose, they can combine Cambridge Pre-U qualifications to achieve the Cambridge Pre-U Diploma; this comprises three Cambridge Pre-U Principal Subjects* together with Global Perspectives and Independent Research (GPR). The Cambridge Pre-U Diploma, therefore, provides the opportunity for interdisciplinary study informed by an international perspective and includes an independent research project.

| first year | second year |
|---|---|
| **CAMBRIDGE PRE-U DIPLOMA** | |
| Cambridge Pre-U Principal Subject | |
| Cambridge Pre-U Principal Subject* | |
| Cambridge Pre-U Principal Subject* | |
| Cambridge Pre-U Global Perspectives and Independent Research (GPR) | |

* Up to two A Levels, Scottish Advanced Highers or IB Diploma programme courses at higher level can be substituted for Principal Subjects.

Learn more about the Cambridge Pre-U Diploma at **www.cambridgeinternational.org/cambridgepreu**

## Support

Cambridge International provides a wide range of support for Pre-U syllabuses, which includes recommended resource lists, teacher guides and example candidate response booklets. Teachers can access these support materials at **www.cambridgeinternational.org/support**

# Syllabus aims

The aims of the syllabus, listed below, are the same for all candidates and are to:

- enable learners to develop a range of mathematical skills and techniques, appreciating their applications in a wide range of contexts, and to apply these techniques to problem-solving in familiar and less familiar contexts
- enable learners to develop an understanding of how different branches of mathematics are connected
- enable learners to recognise how a situation may be represented mathematically and understand how mathematical models can be refined
- encourage learners to use mathematics as an effective means of communication, through the use of correct mathematical language and notation and through the construction of sustained logical arguments, including an appreciation of the limitations of calculator use in relation to obtaining exact solutions.

# Scheme of assessment

For Cambridge Pre-U Mathematics, candidates take all three components.

| Component | | Weighting |
|---|---|---|
| **Paper 1 Pure Mathematics 1**<br><br>Written paper, with structured questions, externally assessed, 80 marks | **2 hours** | $33\frac{1}{3}\%$ |
| **Paper 2 Pure Mathematics 2**<br><br>Written paper, with structured questions, externally assessed, 80 marks | **2 hours** | $33\frac{1}{3}\%$ |
| **Paper 3 Applications of Mathematics**<br><br>Written paper, with structured questions, externally assessed, 80 marks | **2 hours** | $33\frac{1}{3}\%$ |

## Availability

This syllabus is examined in the June examination series.

This syllabus is available to private candidates.

## Combining this with other syllabuses

Candidates can combine this syllabus in a series with any other Cambridge International syllabus, except syllabuses with the same title at the same level.

## Assessment objectives

| | |
|---|---|
| **AO1** | Manipulate mathematical expressions accurately, round answers to an appropriate degree of accuracy, and understand the limitations of solutions obtained using calculators. |
| **AO2** | Construct rigorous mathematical arguments and proofs through the use of precise statements and logical deduction, including extended arguments for problems presented in unstructured form. |
| **AO3** | Recall, select and apply knowledge of mathematical facts, concepts and techniques in a variety of contexts. |
| **AO4** | Understand how mathematics can be used to model situations in the real world and solve problems in relation to both standard models and less familiar contexts, interpreting the results. |

# Relationship between scheme of assessment and assessment objectives

The approximate weightings allocated to each of the assessment objectives (AOs) are summarised below.

### Assessment objectives as a percentage of the qualification

| Assessment objective | Weighting in Pre-U % |
|---|---|
| **AO1** | 43 ± 3 |
| **AO2** | 8 ± 3 |
| **AO3** | 36 ± 3 |
| **AO4** | 13 ± 3 |

### Assessment objectives as a percentage of each component

| Assessment objective | Weighting in components % | | |
|---|---|---|---|
| | Paper 1 | Paper 2 | Paper 3 |
| **AO1** | 46 ± 3 | 46 ± 3 | 38 ± 3 |
| **AO2** | 11 ± 3 | 11 ± 3 | 2 ± 2 |
| **AO3** | 42 ± 3 | 42 ± 3 | 24 ± 3 |
| **AO4** | 1 ± 1 | 1 ± 1 | 37 ± 3 |

# Grading and reporting

Cambridge International Level 3 Pre-U Certificates (Principal Subjects and Global Perspectives Short Course) are qualifications in their own right. Cambridge Pre-U reports achievement on a scale of nine grades: Distinction 1, Distinction 2, Distinction 3, Merit 1, Merit 2, Merit 3, Pass 1, Pass 2 and Pass 3.

| Cambridge Pre-U band | Cambridge Pre-U grade |
|---|---|
| **Distinction** | **1** |
| | **2** |
| | **3** |
| **Merit** | **1** |
| | **2** |
| | **3** |
| **Pass** | **1** |
| | **2** |
| | **3** |

# Grade descriptions

Grade descriptions are provided to give an indication of the standards of achievement likely to have been shown by candidates awarded particular grades. Weakness in one aspect of the examination may be balanced by a better performance in some other aspect.

The following grade descriptions indicate the level of attainment characteristic of the middle of the given grade band.

### Distinction (D2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill.
- If errors are made in candidates' calculations or logic, these are mostly noticed and corrected.
- Candidates make appropriate and efficient use of calculators and other permitted resources, and are aware of any limitations to their use.
- Candidates present results to an appropriate degree of accuracy.
- Candidates use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs.
- When confronted with unstructured problems, candidates can mostly devise and implement an effective solution strategy.
- Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.
- Candidates are usually able to solve problems in less familiar contexts.
- Candidates correctly refer results from calculations using a mathematical model to the original situation; they give sensible interpretations of their results in context and mostly make sensible comments or predictions.
- Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world.
- Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts.
- Candidates make intelligent comments on any modelling assumptions.

### Merit (M2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill.
- Candidates often notice and correct errors in their calculations.
- Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use.
- Candidates usually present results to an appropriate degree of accuracy.
- Candidates use mathematical language with some skill and usually proceed logically through extended arguments or proofs.
- When confronted with unstructured problems, candidates usually devise and implement an effective solution strategy.
- Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.
- Candidates are often able to solve problems in less familiar contexts.

- Candidates usually correctly refer results from calculations using a mathematical model to the original situation; they usually give sensible interpretations of their results in context and sometimes make sensible comments or predictions.
- Candidates recall or recognise most of the standard models that are needed, and usually select appropriate ones to represent a variety of situations in the real world.
- Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts.
- Candidates often make intelligent comments on any modelling assumptions.

### Pass (P2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill.
- If errors are made in candidates' calculations or logic, these are sometimes noticed and corrected.
- Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use.
- Candidates often present results to an appropriate degree of accuracy.
- Candidates frequently use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.
- When confronted with unstructured problems, candidates can sometimes make some headway with an effective solution strategy.
- Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.
- Candidates try to solve problems in less familiar contexts.
- Candidates frequently correctly refer results from calculations using a mathematical model to the original situation; they sometimes interpret their results in context and attempt to make sensible comments or predictions.
- Candidates recall or recognise some of the standard models that are needed, and frequently select appropriate ones to represent a variety of situations in the real world.
- Candidates frequently comprehend or understand the meaning of translations into mathematics of common realistic contexts.

# Description of components

For all components, knowledge of the content of GCSE/IGCSE™ or O Level Mathematics is assumed.

## Paper 1 and Paper 2 Pure Mathematics

Written papers, 2 hours, 80 marks each

Paper 1 and Paper 2 may contain questions on any topics from the pure mathematics syllabus content:

- quadratics
- algebra
- functions
- coordinate geometry
- circular measure
- trigonometry
- sequences and series
- logarithms and exponentials
- differentiation
- integration
- vector geometry
- differential equations
- complex numbers
- numerical methods

These papers will consist of a mixture of short, medium and longer questions. Each component will have a total of 80 marks. In addition to the topics listed, candidates will be expected to apply their knowledge of pure mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

## Paper 3 Applications of Mathematics

Written paper, 2 hours, 80 marks

- Probability
  - analysis of data
  - probability laws
  - permutations and combinations
  - discrete random variables
  - the normal distribution

- Mechanics
  - kinematics of motion in a straight line
  - force and equilibrium
  - friction
  - Newton's laws of motion
  - linear momentum and impulse
  - motion of a projectile

The paper will consist of a mixture of medium and longer questions, with a total of 80 marks, of which approximately 40 marks will relate to probability and approximately 40 marks to mechanics. In addition to the topics listed, candidates will be expected to apply their knowledge of pure mathematics to questions. Candidates will be expected to answer all questions.

## Use of calculators

The use of scientific calculators will be permitted in all components. Graphic calculators will not be permitted. Candidates will be expected to be aware of the limitations inherent in the use of calculators.

## Mathematical tables and formulae

Candidates will be provided with a booklet of mathematical formulae and tables for use in the examination.

# Syllabus content

This syllabus gives you the flexibility to design a course that will interest, challenge and engage your learners. Where appropriate you are responsible for selecting suitable subject contexts, resources and examples to support your learners' study. These should be appropriate for the learners' age, cultural background and learning context as well as complying with your school policies and local legal requirements.

## Paper 1 and Paper 2 Pure Mathematics

Throughout, candidates should be familiar with the logical processes and conventional symbolic machinery involved in a mathematical development or proof by direct argument. They should also understand the methods of proof by exhaustion and contradiction, and of disproof by counterexample.

**PURE MATHEMATICS**

### Quadratics

Candidates should be able to:

- carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$, and understand the relationship between this form and the graph of $y = ax^2 + bx + c$
- find the discriminant of a quadratic polynomial $ax^2 + bx + c$, and understand how this relates to the number of real roots of the equation $ax^2 + bx + c = 0$
- manipulate expressions involving surds
- solve quadratic equations, and linear and quadratic inequalities, in one unknown
- solve, by substitution, a pair of simultaneous equations, of which one is linear and the other is quadratic
- recognise and solve equations that are quadratic in some function.

### Algebra

Candidates should be able to:

- understand the meaning of $|x|$, including sketching the graph of $y = |\mathrm{f}(x)|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ in the course of solving equations and inequalities which may also include equations in the form $|x - a| = bx + c$
- divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and the remainder (which may be zero)
- use the factor theorem and the remainder theorem
- recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than:
  - $(ax + b)(cx + d)(ex + f)$
  - $(ax + b)(cx + d)^2$
  - $(ax + b)(c^2x^2 + d^2)$

  and where the degree of the numerator is less than that of the denominator.

## Functions

Candidates should be able to:

- understand the terms function, domain, codomain, range, one-one function, inverse function and composite function
- identify the range of a given function in simple cases, and find the composite of two given functions
- determine whether a function is one-one, and find the inverse of a one-one function in simple cases
- understand the relationship between the graphs of a one-one function and its inverse.

## Coordinate geometry

Candidates should be able to:

- find the length, gradient and mid-point of a line segment, given the coordinates of the end points
- find the equation of a straight line, given sufficient information (e.g. two points, or one point and the gradient)
- understand and use the relationships between the gradients of parallel and perpendicular lines
- interpret and use linear equations in context
- understand the relationship between a graph and its associated algebraic equation, including asymptotes parallel to the coordinate axes in simple cases, and use the relationship between points of intersection of graphs and solutions of equations (including, for simple cases, the relationship between tangents and repeated roots)
- recognise and use the equation of a circle
- understand and use the transformations of graphs given by $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(ax)$, $y = -\mathrm{f}(x)$, $y = \mathrm{f}(-x)$ and simple combinations of these.

## Circular measure

Candidates should be able to:

- understand the definition of radian measure and use the relationship between radians and degrees
- use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle.

## Trigonometry

Candidates should be able to:

- sketch and use the graphs of the sine, cosine and tangent functions (for angles of any size and using degrees or radians), e.g. $y = \sin 3x$, or $y = \cos (x + 30°)$
- use the exact values of sine, cosine and tangent of 30°, 45°, 60° and related angles
- solve problems using the sine and cosine rules and use the formula $\frac{1}{2}ab\sin C$ for the area of a triangle
- find all the solutions, lying in specified intervals, of simple trigonometric equations, e.g. solve $2 \tan 3\theta = -1$, for $0 \leqslant \theta < 2\pi$
- understand the use of $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$ to denote the principal values of the inverse trigonometric functions
- understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude

- use trigonometric identities for the simplification of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of:
  - $\frac{\sin\theta}{\cos\theta} = \tan\theta$ and $\sin^2\theta + \cos^2\theta = 1$
  - $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$
  - the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
  - the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$
  - the expressions of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.

## Sequences and series

Candidates should be able to:

- use formulae for the $n$th term of a sequence, and for the sum of a finite series
- understand and use the sigma notation
- recognise arithmetic and geometric progressions
- use the formulae for the $n$th term and sum of the first $n$ terms to solve problems involving arithmetic and geometric progressions
- use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression
- use the binomial expansion of:
  - $(a + b)^n$, where $n$ is a positive integer
  - $(1 + x)^n$, where $n$ is a rational number and $|x| < 1$
    and be able to adapt this method to the expansion of expressions of the form $(a + bx^c)^n$, stating the range of values for which such an expansion is valid
- understand the use of recurrence relations in defining some sequences, and recognise alternating, periodic, convergent and divergent sequences.

## Logarithms and exponentials

Candidates should be able to:

- understand the relationship between logarithms and indices, and use the laws of logarithms
- use logarithms, for example to solve equations of the form $a^x = b$, and related equations or inequalities such as $a^{2x} + pa^x + q = 0$
- understand the definitions and properties of $e^x$ and $\ln x$, including their relationship as inverse functions and their graphs
- use logarithms to transform appropriate relationships to linear form.

## Differentiation

Candidates should be able to:

- understand the idea of the gradient of a curve, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
- understand the idea of differentiation from first principles, and be able to carry it out for $x^n$, where $n$ is a positive or negative integer or $\pm\frac{1}{2}$
- use the derivatives of $x^n$ (for any rational $n$), $e^x$, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums and differences
- apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change
- locate stationary points, and determine by calculation whether the points are local maximum or minimum points (the ability to distinguish between maximum points and minimum points is required, but identification of points of inflexion is not included)
- use the chain rule for differentiation
- differentiate products and quotients
- determine the first and second derivatives of a function defined either parametrically or implicitly
- sketch the graphs of functions defined parametrically or implicitly in simple cases, and find the cartesian equation of a function defined parametrically.

## Integration

Candidates should be able to:

- understand integration as the reverse process of differentiation, and integrate $x^n$ (for any rational $n$), $e^x$, $\sin x$, $\cos x$ and $\sec^2 x$, together with constant multiples, sums and differences
- solve problems involving the evaluation of a constant of integration
- use definite integration to find the area of a region bounded by a curve and lines parallel to the axes, or between two curves
- use trigonometric relationships (such as double-angle formulae) to integrate functions such as $\cos^2 x$
- integrate rational functions by means of decomposition into partial fractions
- recognise and integrate an integrand of the form $\frac{kf'(x)}{f(x)}$
- use integration by parts to integrate suitable products
- use substitution to simplify and evaluate definite or indefinite integrals
- calculate a volume of revolution about the $x$- or $y$-axis, to include the volume of a solid generated by rotating a region between two curves.

## Vector geometry

Candidates should be able to:

- use standard notations for vectors, and be familiar with the terms *position vector* and *displacement vector*
- carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometric terms
- calculate the magnitude of a vector
- calculate the scalar product of two vectors, and use the scalar product to determine the angle between two vectors and to solve problems involving perpendicular vectors

- understand and use the vector form of a straight line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, and understand and use the equivalent cartesian form of a straight line
- determine whether two lines are parallel, intersect or are skew
- find the angle between two lines, and the point of intersection of two lines when it exists.

## Differential equations

Candidates should be able to:

- formulate a simple statement involving a rate of change as a differential equation introducing, if necessary, a constant of proportionality, and find a general solution for the differential equation
- find, by integration, a general solution for a first-order differential equation in which the variables are separable
- use initial conditions to find a particular solution
- interpret the solution of a differential equation in the context of a problem being modelled by the equation.

## Complex numbers

Candidates should be able to:

- understand the definition of a complex number, and the meaning of the terms real part, imaginary part, modulus, argument and conjugate; use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal
- carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form $x + \mathrm{i}y$
- find the complex roots of a quadratic equation with real coefficients, or of a cubic equation with real coefficients where one real factor can be identified by the factor theorem
- recognise that complex roots of polynomial equations occur in conjugate pairs when the coefficients are real
- represent complex numbers geometrically by means of an Argand diagram, and indicate loci no more complicated than, e.g. $|z - a| = |z - b|$.

## Numerical methods

Candidates should be able to:

- investigate the location of the roots of $\mathrm{f}(x) = 0$ by a change of sign of $y = \mathrm{f}(x)$ over an interval
- implement the direct iteration method for the numerical evaluation of an approximation to a root of $x = \mathrm{F}(x)$, understand and recognise informally the relationship between the magnitude of the derivative of F at the root and the convergence or divergence of the iterative scheme
- implement the Newton-Raphson iteration method for the numerical evaluation of an approximation to a root of $\mathrm{f}(x) = 0$, and understand the geometric derivation of this method
- understand the concept of rate of convergence of an iterative scheme, and in particular how this concept is realised for the schemes in the second and third bullet points of this section.

# Paper 3 Applications of Mathematics

**PROBABILITY**

## Analysis of data

Candidates should be able to:

- use and interpret different measures of central tendency (mean, median and mode) and variation (range, interquartile range and standard deviation), e.g. in comparing and contrasting sets of data
- understand and use the effect of linear transformations on mean and standard deviation
- calculate the mean, standard deviation and variance from raw data or summary statistics
- identify outliers (using the $1.5 \times$ IQR and $\pm 2$ standard deviation criteria) and describe whether a set of data has positive or negative skew
- understand the concepts of dependent and independent variables, linear correlation and regression lines for bivariate data, and understand and use the term residual
- construct and use scatter graphs
- use the product-moment correlation coefficient as a measure of correlation, and use covariance and variance in the construction of regression lines.

## Probability laws

Candidates should be able to:

- evaluate probabilities in simple cases by calculation using permutations and combinations
- understand and use Venn diagrams for up to three events
- understand the meaning of exclusive and independent events, and calculate and use conditional probabilities in simple cases, e.g. situations that can be represented by means of a tree diagram
- use the notation $\mathrm{P}(A)$ for the probability of the event $A$, and the notations $\mathrm{P}(A \cup B)$, $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A|B)$ relating to probabilities involving two events
- understand and use the result $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
- use the result $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B|A) = \mathrm{P}(B)\mathrm{P}(A|B)$ to solve problems involving conditional probability.

## Permutations and combinations

Candidates should be able to:

- solve problems about selections, e.g. finding the number of ways in which a team of three men and two women can be selected from a group of six men and five women
- solve problems about arrangements of objects in a line, including those involving:
  - repetition (e.g. the number of ways of arranging the letters of the word 'STATISTICS')
  - restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).

## Discrete random variables

Candidates should be able to:

- use formulae for probabilities for the binomial and geometric distributions, model given situations by one of these as appropriate, and recognise the notations $B(n, p)$ and $Geo(p)$
- use tables of cumulative binomial probabilities
- construct a probability distribution table relating to a given situation involving a discrete random variable $X$, and calculate the expectation, variance and standard deviation of $X$
- use formulae for the expectation and variance of the binomial and geometric distributions.

## The normal distribution

Candidates should be able to:

- understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables
- solve problems concerning a variable $X$, where $X \sim N(\mu, \sigma^2)$, including:
  - using given values of $\mu$ and $\sigma$ to find the value of $P(X < x)$, or a related probability, or conversely to find the relevant value of $x$
  - finding the values of $\mu$ and $\sigma$ from given probabilities.

**MECHANICS**

## Kinematics of motion in a straight line

Candidates should be able to:

- recognise distance and speed as scalar quantities, and displacement, velocity and acceleration as vector quantities
- sketch and interpret displacement-time and velocity-time graphs, and use the facts that:
  - the area under a $v$-$t$ graph represents displacement
  - the gradient of an $x$-$t$ graph represents velocity
  - the gradient of a $v$-$t$ graph represents acceleration
- use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration
- use the formulae for motion with constant acceleration
- extend knowledge to include motion in two or three dimensions.

## Force and equilibrium

Candidates should be able to:

- understand the vector nature of force, and use directed line segments to represent forces
- calculate the resultant of two or more forces acting at a point, and use vector addition to solve problems involving resultants and components of forces
- find and use perpendicular components of a force, e.g. in finding the resultant of a system of forces, or to calculate the magnitude and direction of a force
- identify the forces acting in a given situation, including weight, tension, normal reaction and friction
- use the principle that a particle is in equilibrium if and only if the vector sum of the forces acting is zero, or equivalently if and only if the sum of the resolved parts in any given direction is zero
- use the model of a smooth contact and understand the limitations of the model.

## Friction

Candidates should be able to:

- represent the contact force between two rough surfaces by two components: the *normal force* and the *frictional force*
- recall the definition of coefficient of friction
- understand the concept of limiting friction and the relation $F \leqslant \mu R$
- use the friction law in problems that involve equilibrium of a particle or the use of Newton's laws.

## Newton's laws of motion

Candidates should be able to:

- apply Newton's laws of motion to the linear motion of bodies of constant mass moving under the action of constant forces
- model the motion of a body moving vertically or on an inclined plane as motion with constant acceleration, and understand any limitations of this model
- solve simple problems which may be modelled as the motion of two or three connected particles, for example a car towing a caravan, or two objects connected by a light inextensible string passing over a fixed smooth peg or light pulley
- use Newton's third law, for example to calculate the reaction on a particle in an accelerating lift.

## Linear momentum and impulse

Candidates should be able to:

- understand the vector nature of linear momentum of a particle or system of particles moving in a straight line
- understand the concept of the impulse of a force causing a change in linear momentum, either where a constant force acts for a finite time or where an instantaneous collision occurs
- understand and use conservation of linear momentum in simple applications involving the direct impact of two bodies moving in the same straight line, including the case where the bodies coalesce
- understand and use Newton's law of restitution in the course of solving one-dimensional problems that may be modelled as the direct impact of two spheres, or as the direct impact of a sphere with a fixed plane surface.

## Motion of a projectile

Candidates should be able to:

- model the motion of a projectile as a particle moving with constant acceleration, and understand any limitations of this model
- use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached
- derive and use the cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.

## Mathematical formulae and statistical tables

Examinations for this syllabus may use relevant formulae from the following list.

**PURE MATHEMATICS**

**Mensuration**

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

**Trigonometry**

$a^2 = b^2 + c^2 - 2bc\cos A$

**Arithmetic series**

$u_n = a + (n-1)d$

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

**Geometric series**

$u_n = ar^{n-1}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

**Summations**

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

**Binomial series**

$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n, (n \in \mathbb{N})$, where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots (|x| < 1, n \in \mathbb{R})$

**Logarithms and exponentials**

$e^{x\ln a} = a^x$

**Complex numbers**

$\{r(\cos\theta + i\sin\theta)\}^n = r^n(\cos n\theta + i\sin n\theta)$

$e^{i\theta} = \cos\theta + i\sin\theta$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k i}{n}}$, for $k = 0, 1, 2, ..., n-1$

**Maclaurin's series**

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1} \frac{x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 \leqslant x \leqslant 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 < x < 1)$$

**Hyperbolic functions**

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\{x + \sqrt{x^2 - 1}\} \quad (x \geqslant 1)$$

$$\sinh^{-1} x = \ln\{x + \sqrt{x^2 + 1}\}$$

$$\tanh^{-1} x = \tfrac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$$

**Coordinate geometry**

The perpendicular distance from $(h, k)$ to $ax + by + c = 0$ is $\dfrac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients $m_1$ and $m_2$ is $\tan^{-1} \left| \dfrac{m_1 - m_2}{1 + m_1 m_2} \right|$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

For $t = \tan \tfrac{1}{2} A$: $\sin A = \dfrac{2t}{1+t^2}$, $\cos A = \dfrac{1-t^2}{1+t^2}$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

### Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

The point dividing $AB$ in the ratio $\lambda:\mu$ is $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product: $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}=\begin{vmatrix}\mathbf{i} & a_1 & b_1\\ \mathbf{j} & a_2 & b_2\\ \mathbf{k} & a_3 & b_3\end{vmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\ a_3b_1-a_1b_3\\ a_1b_2-a_2b_1\end{pmatrix}$

If $A$ is the point with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, then the straight line through $A$ with direction vector **b** has cartesian equation

$$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}\ (=\lambda)$$

The plane through $A$ with normal vector $\mathbf{n}=n_1\mathbf{i}+n_2\mathbf{j}+n_3\mathbf{k}$ has cartesian equation

$$n_1x+n_2y+n_3z+d=0 \text{ where } d=-\mathbf{a}.\mathbf{n}$$

The plane through non-collinear points $A$, $B$ and $C$ has vector equation

$$\mathbf{r}=\mathbf{a}+\lambda(\mathbf{b}-\mathbf{a})+\mu(\mathbf{c}-\mathbf{a})=(1-\lambda-\mu)\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$

The perpendicular distance of $(\alpha,\beta,\gamma)$ from $n_1x+n_2y+n_3z+d=0$ is $\dfrac{|n_1\alpha+n_2\beta+n_3\gamma+d|}{\sqrt{n_1^2+n_2^2+n_3^2}}$

### Matrix transformations

Anticlockwise rotation through $\theta$ about $O$: $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$

Reflection in the line $y=(\tan\theta)x$: $\begin{pmatrix}\cos2\theta & \sin2\theta\\ \sin2\theta & -\cos2\theta\end{pmatrix}$

### Differentiation

| **f($x$)** | **f ′($x$)** |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sin^{-1}x$ | $\dfrac{1}{\sqrt{1-x^2}}$ |
| $\cos^{-1}x$ | $-\dfrac{1}{\sqrt{1-x^2}}$ |
| $\tan^{-1}x$ | $\dfrac{1}{1+x^2}$ |
| $\sec x$ | $\sec x\tan x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x\cot x$ |
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |
| $\tanh x$ | $\operatorname{sech}^2 x$ |
| $\sinh^{-1}x$ | $\dfrac{1}{\sqrt{1+x^2}}$ |
| $\cosh^{-1}x$ | $\dfrac{1}{\sqrt{x^2-1}}$ |
| $\tanh^{-1}x$ | $\dfrac{1}{1-x^2}$ |

**Integration** (+ constant; $a > 0$ where relevant)

| $\mathrm{f}(x)$ | $\int \mathrm{f}(x)\,\mathrm{d}x$ |
|---|---|
| $\sec^2 kx$ | $\frac{1}{k}\tan kx$ |
| $\tan x$ | $\ln\lvert\sec x\rvert$ |
| $\cot x$ | $\ln\lvert\sin x\rvert$ |
| $\operatorname{cosec} x$ | $-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\tfrac{1}{2}x)\rvert$ |
| $\sec x$ | $\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$ |
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |
| $\tanh x$ | $\ln\cosh x$ |
| $\frac{1}{\sqrt{a^2 - x^2}}$ | $\sin^{-1}\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$ |
| $\frac{1}{a^2 + x^2}$ | $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$ |
| $\frac{1}{\sqrt{x^2 - a^2}}$ | $\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 - a^2}\right\} \quad (x > a)$ |
| $\frac{1}{\sqrt{a^2 + x^2}}$ | $\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 + a^2}\right\}$ |
| $\frac{1}{a^2 - x^2}$ | $\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$ |
| $\frac{1}{x^2 - a^2}$ | $\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$ |

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

**Area of a sector**

$A = \frac{1}{2}\int r^2\,\mathrm{d}\theta$ (polar coordinates)

$A = \frac{1}{2}\int\left(x\frac{\mathrm{d}y}{\mathrm{d}t} - y\frac{\mathrm{d}x}{\mathrm{d}t}\right)\mathrm{d}t$ (parametric form)

**Arc length**

$s = \int\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ (cartesian coordinates)

$s = \int\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$ (parametric form)

$s = \int\sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ (polar form)

**Surface area of revolution**

$S_x = 2\pi\int y\,\mathrm{d}s$

$S_y = 2\pi\int x\,\mathrm{d}s$

**Numerical solution of equations**

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

## MECHANICS

**Motion in a circle**

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\dfrac{v^2}{r}$

## PROBABILITY

**Probability**

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B|A)$

$$\mathrm{P}(A|B) = \frac{\mathrm{P}(B|A)\mathrm{P}(A)}{\mathrm{P}(B|A)\mathrm{P}(A) + \mathrm{P}(B|A')\mathrm{P}(A')}$$

Bayes' Theorem: $\mathrm{P}(A_j|B) = \dfrac{\mathrm{P}(A_j)\mathrm{P}(B|A_j)}{\sum \mathrm{P}(A_i)\mathrm{P}(B|A_i)}$

**Discrete distributions**

For a discrete random variable $X$ taking values $x_i$ with probabilities $p_i$

Expectation (mean): $\mathrm{E}(X) = \mu = \sum x_i p_i$

Variance: $\mathrm{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \sum \mathrm{g}(x_i) p_i$

The probability generating function (P.G.F.) of $X$ is $\mathrm{G}_X(t) = \mathrm{E}(t^X)$,

and $\mathrm{E}(X) = \mathrm{G}'_X(1)$, $\mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \{\mathrm{G}'_X(1)\}^2$

For $Z = X + Y$, where $X$ and $Y$ are independent: $\mathrm{G}_Z(t) = \mathrm{G}_X(t)\mathrm{G}_Y(t)$

The moment generating function (M.G.F.) of $X$ is $\mathrm{M}_X(t) = \mathrm{E}(\mathrm{e}^{tX})$,

and $\mathrm{E}(X) = \mathrm{M}'_X(0)$, $E(X^n) = M_X^{(n)}(0)$, $\mathrm{Var}(X) = \mathrm{M}''_X(0) - \{\mathrm{M}'_X(0)\}^2$

For $Z = X + Y$, where $X$ and $Y$ are independent: $\mathrm{M}_Z(t) = \mathrm{M}_X(t)\mathrm{M}_Y(t)$

**Standard discrete distributions**

| Distribution of $X$ | $\mathrm{P}(X = x)$ | Mean | Variance | P.G.F. | M.G.F. |
|---|---|---|---|---|---|
| Binomial $\mathrm{B}(n, p)$ | $\binom{n}{x} p^x (1-p)^{n-x}$ | $np$ | $np(1-p)$ | $(1 - p + pt)^n$ | $(1 - p + p\mathrm{e}^t)^n$ |
| Poisson $\mathrm{Po}(\lambda)$ | $\mathrm{e}^{-\lambda} \frac{\lambda^x}{x!}$ | $\lambda$ | $\lambda$ | $\mathrm{e}^{\lambda(t-1)}$ | $\mathrm{e}^{\lambda(\mathrm{e}^t - 1)}$ |
| Geometric $\mathrm{Geo}(p)$ on 1, 2, ... | $p(1-p)^{x-1}$ | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ | $\frac{pt}{1-(1-p)t}$ | $\frac{p\mathrm{e}^t}{1-(1-p)\mathrm{e}^t}$ |

**Continuous distributions**

For a continuous random variable $X$ having probability density function (P.D.F.) f

Expectation (mean): $\mathrm{E}(X)=\mu=\int x\,\mathrm{f}(x)\,\mathrm{d}x$

Variance: $\mathrm{Var}(X)=\sigma^2=\int (x-\mu)^2\,\mathrm{f}(x)\,\mathrm{d}x=\int x^2\,\mathrm{f}(x)\,\mathrm{d}x-\mu^2$

For a function $\mathrm{g}(X)$ : $\mathrm{E}(\mathrm{g}(X))=\int \mathrm{g}(x)\,\mathrm{f}(x)\,\mathrm{d}x$

Cumulative distribution function: $\mathrm{F}(x)=\mathrm{P}(X\leqslant x)=\int_{-\infty}^{x}\mathrm{f}(t)\,\mathrm{d}t$

The moment generating function (M.G.F.) of $X$ is $\mathrm{M}_X(t)=\mathrm{E}(\mathrm{e}^{tX})$,

and $\mathrm{E}(X)=\mathrm{M}'_X(0)$, $E(X^n)=M_X^{(n)}(0)$, $\mathrm{Var}(X)=\mathrm{M}''_X(0)-\{\mathrm{M}'_X(0)\}^2$

For $Z=X+Y$, where $X$ and $Y$ are independent: $\mathrm{M}_Z(t)=\mathrm{M}_X(t)\mathrm{M}_Y(t)$

**Standard continuous distributions**

| Distribution of $X$ | P.D.F. | Mean | Variance | M.G.F. |
|---|---|---|---|---|
| Uniform (Rectangular) on $[a, b]$ | $\frac{1}{b-a}$ | $\frac{1}{2}(a+b)$ | $\frac{1}{12}(b-a)^2$ | $\frac{\mathrm{e}^{bt}-\mathrm{e}^{at}}{(b-a)t}$ |
| Exponential | $\lambda\mathrm{e}^{-\lambda x}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | $\frac{\lambda}{\lambda-t}$ |
| Normal $\mathrm{N}(\mu,\sigma^2)$ | $\frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ | $\mu$ | $\sigma^2$ | $\mathrm{e}^{\mu t+\frac{1}{2}\sigma^2t^2}$ |

**Expectation algebra**

For independent random variables $X$ and $Y$

$\mathrm{E}(XY)=\mathrm{E}(X)\,\mathrm{E}(Y)$, $\mathrm{Var}\,(aX\pm bY)=a^2\,\mathrm{Var}(X)+b^2\,\mathrm{Var}\,(Y)$

**Sampling distributions**

For a random sample $X_1, X_2, \ldots, X_n$ of $n$ independent observations from a distribution having mean $\mu$ and variance $\sigma^2$

$\overline{X}$ is an unbiased estimator of $\mu$, with $\mathrm{Var}(\overline{X})=\frac{\sigma^2}{n}$

$S^2$ is an unbiased estimator of $\sigma^2$, where $S^2=\frac{\Sigma(X_i-\overline{X})^2}{n-1}$

For a random sample of $n$ observations from $\mathrm{N}(\mu,\sigma^2)$

$\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\sim\mathrm{N}(0,1)$

$\frac{\overline{X}-\mu}{S/\sqrt{n}}\sim t_{n-1}$ (also valid in matched-pairs situations)

If $X$ is the observed number of successes in $n$ independent Bernoulli trials, in each of which the probability of success is $p$, and $Y=\frac{X}{n}$, then $\mathrm{E}(Y)=p$ and $\mathrm{Var}(Y)=\frac{p(1-p)}{n}$

For a random sample of $n_x$ observations from $\mathrm{N}(\mu_x, \sigma_x^2)$ and, independently, a random sample of $n_y$ observations from $\mathrm{N}(\mu_y, \sigma_y^2)$

$$\frac{(\overline{X}-\overline{Y})-(\mu_x-\mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x}+\dfrac{\sigma_y^2}{n_y}}} \sim \mathrm{N}(0,1)$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown), then $\dfrac{(\overline{X}-\overline{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\dfrac{1}{n_x}+\dfrac{1}{n_y}\right)}} \sim t_{n_x+n_y-2}$, where $S_p^2 = \dfrac{(n_x-1)S_x^2+(n_y-1)S_y^2}{n_x+n_y-2}$

**Correlation and regression**

For a set of $n$ pairs of values $(x_i, y_i)$

$$S_{xx} = \Sigma(x_i-\overline{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i-\overline{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i-\overline{x})(y_i-\overline{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product-moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\{\Sigma(x_i-\overline{x})^2\}\{\Sigma(y_i-\overline{y})^2\}}} = \frac{\Sigma x_i y_i - \dfrac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \dfrac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of $y$ on $x$ is $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\Sigma(x_i-\overline{x})(y_i-\overline{y})}{\Sigma(x_i-\overline{x})^2}$

Least squares regression line of $y$ on $x$ is $y = a + bx$ where $a = \overline{y} - b\overline{x}$

# CUMULATIVE BINOMIAL PROBABILITIES

$$P(X \leqslant x) = \sum_{r=0}^{x} {}^{n}C_r \,(1-p)^{n-r} p^r$$

| $n = 5$<br>$p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.7738 | 0.5905 | 0.4437 | 0.4019 | 0.3277 | 0.2373 | 0.1681 | 0.1317 | 0.1160 | 0.0778 | 0.0503 | 0.0313 | 0.0185 | 0.0102 | 0.0053 | 0.0041 | 0.0024 | 0.0010 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 1 | 0.9774 | 0.9185 | 0.8352 | 0.8038 | 0.7373 | 0.6328 | 0.5282 | 0.4609 | 0.4284 | 0.3370 | 0.2562 | 0.1875 | 0.1312 | 0.0870 | 0.0540 | 0.0453 | 0.0308 | 0.0156 | 0.0067 | 0.0033 | 0.0022 | 0.0005 | 0.0000 |
| 2 | 0.9988 | 0.9914 | 0.9734 | 0.9645 | 0.9421 | 0.8965 | 0.8369 | 0.7901 | 0.7648 | 0.6826 | 0.5931 | 0.5000 | 0.4069 | 0.3174 | 0.2352 | 0.2099 | 0.1631 | 0.1035 | 0.0579 | 0.0355 | 0.0266 | 0.0086 | 0.0012 |
| 3 | 1.0000 | 0.9995 | 0.9978 | 0.9967 | 0.9933 | 0.9844 | 0.9692 | 0.9547 | 0.9460 | 0.9130 | 0.8688 | 0.8125 | 0.7438 | 0.6630 | 0.5716 | 0.5391 | 0.4718 | 0.3672 | 0.2627 | 0.1962 | 0.1648 | 0.0815 | 0.0226 |
| 4 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9990 | 0.9976 | 0.9959 | 0.9947 | 0.9898 | 0.9815 | 0.9688 | 0.9497 | 0.9222 | 0.8840 | 0.8683 | 0.8319 | 0.7627 | 0.6723 | 0.5981 | 0.5563 | 0.4095 | 0.2262 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $n = 6$<br>$p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.7351 | 0.5314 | 0.3771 | 0.3349 | 0.2621 | 0.1780 | 0.1176 | 0.0878 | 0.0754 | 0.0467 | 0.0277 | 0.0156 | 0.0083 | 0.0041 | 0.0018 | 0.0014 | 0.0007 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.9672 | 0.8857 | 0.7765 | 0.7368 | 0.6554 | 0.5339 | 0.4202 | 0.3512 | 0.3191 | 0.2333 | 0.1636 | 0.1094 | 0.0692 | 0.0410 | 0.0223 | 0.0178 | 0.0109 | 0.0046 | 0.0016 | 0.0007 | 0.0004 | 0.0001 | 0.0000 |
| 2 | 0.9978 | 0.9842 | 0.9527 | 0.9377 | 0.9011 | 0.8306 | 0.7443 | 0.6804 | 0.6471 | 0.5443 | 0.4415 | 0.3438 | 0.2553 | 0.1792 | 0.1174 | 0.1001 | 0.0705 | 0.0376 | 0.0170 | 0.0087 | 0.0059 | 0.0013 | 0.0001 |
| 3 | 0.9999 | 0.9987 | 0.9941 | 0.9913 | 0.9830 | 0.9624 | 0.9295 | 0.8999 | 0.8826 | 0.8208 | 0.7447 | 0.6563 | 0.5585 | 0.4557 | 0.3529 | 0.3196 | 0.2557 | 0.1694 | 0.0989 | 0.0623 | 0.0473 | 0.0159 | 0.0022 |
| 4 | 1.0000 | 0.9999 | 0.9996 | 0.9993 | 0.9984 | 0.9954 | 0.9891 | 0.9822 | 0.9777 | 0.9590 | 0.9308 | 0.8906 | 0.8364 | 0.7667 | 0.6809 | 0.6488 | 0.5798 | 0.4661 | 0.3446 | 0.2632 | 0.2235 | 0.1143 | 0.0328 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9993 | 0.9986 | 0.9982 | 0.9959 | 0.9917 | 0.9844 | 0.9723 | 0.9533 | 0.9246 | 0.9122 | 0.8824 | 0.8220 | 0.7379 | 0.6651 | 0.6229 | 0.4686 | 0.2649 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $n = 7$<br>$p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.6983 | 0.4783 | 0.3206 | 0.2791 | 0.2097 | 0.1335 | 0.0824 | 0.0585 | 0.0490 | 0.0280 | 0.0152 | 0.0078 | 0.0037 | 0.0016 | 0.0006 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.9556 | 0.8503 | 0.7166 | 0.6698 | 0.5767 | 0.4449 | 0.3294 | 0.2634 | 0.2338 | 0.1586 | 0.1024 | 0.0625 | 0.0357 | 0.0188 | 0.0090 | 0.0069 | 0.0038 | 0.0013 | 0.0004 | 0.0001 | 0.0001 | 0.0000 | 0.0000 |
| 2 | 0.9962 | 0.9743 | 0.9262 | 0.9042 | 0.8520 | 0.7564 | 0.6471 | 0.5706 | 0.5323 | 0.4199 | 0.3164 | 0.2266 | 0.1529 | 0.0963 | 0.0556 | 0.0453 | 0.0288 | 0.0129 | 0.0047 | 0.0020 | 0.0012 | 0.0002 | 0.0000 |
| 3 | 0.9998 | 0.9973 | 0.9879 | 0.9824 | 0.9667 | 0.9294 | 0.8740 | 0.8267 | 0.8002 | 0.7102 | 0.6083 | 0.5000 | 0.3917 | 0.2898 | 0.1998 | 0.1733 | 0.1260 | 0.0706 | 0.0333 | 0.0176 | 0.0121 | 0.0027 | 0.0002 |
| 4 | 1.0000 | 0.9998 | 0.9988 | 0.9980 | 0.9953 | 0.9871 | 0.9712 | 0.9547 | 0.9444 | 0.9037 | 0.8471 | 0.7734 | 0.6836 | 0.5801 | 0.4677 | 0.4294 | 0.3529 | 0.2436 | 0.1480 | 0.0958 | 0.0738 | 0.0257 | 0.0038 |
| 5 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9996 | 0.9987 | 0.9962 | 0.9931 | 0.9910 | 0.9812 | 0.9643 | 0.9375 | 0.8976 | 0.8414 | 0.7662 | 0.7366 | 0.6706 | 0.5551 | 0.4233 | 0.3302 | 0.2834 | 0.1497 | 0.0444 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9995 | 0.9994 | 0.9984 | 0.9963 | 0.9922 | 0.9848 | 0.9720 | 0.9510 | 0.9415 | 0.9176 | 0.8665 | 0.7903 | 0.7209 | 0.6794 | 0.5217 | 0.3017 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $n = 8$<br>$p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.6634 | 0.4305 | 0.2725 | 0.2326 | 0.1678 | 0.1001 | 0.0576 | 0.0390 | 0.0319 | 0.0168 | 0.0084 | 0.0039 | 0.0017 | 0.0007 | 0.0002 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.9428 | 0.8131 | 0.6572 | 0.6047 | 0.5033 | 0.3671 | 0.2553 | 0.1951 | 0.1691 | 0.1064 | 0.0632 | 0.0352 | 0.0181 | 0.0085 | 0.0036 | 0.0026 | 0.0013 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9942 | 0.9619 | 0.8948 | 0.8652 | 0.7969 | 0.6785 | 0.5518 | 0.4682 | 0.4278 | 0.3154 | 0.2201 | 0.1445 | 0.0885 | 0.0498 | 0.0253 | 0.0197 | 0.0113 | 0.0042 | 0.0012 | 0.0004 | 0.0002 | 0.0000 | 0.0000 |
| 3 | 0.9996 | 0.9950 | 0.9786 | 0.9693 | 0.9437 | 0.8862 | 0.8059 | 0.7414 | 0.7064 | 0.5941 | 0.4770 | 0.3633 | 0.2604 | 0.1737 | 0.1061 | 0.0879 | 0.0580 | 0.0273 | 0.0104 | 0.0046 | 0.0029 | 0.0004 | 0.0000 |
| 4 | 1.0000 | 0.9996 | 0.9971 | 0.9954 | 0.9896 | 0.9727 | 0.9420 | 0.9121 | 0.8939 | 0.8263 | 0.7396 | 0.6367 | 0.5230 | 0.4059 | 0.2936 | 0.2586 | 0.1941 | 0.1138 | 0.0563 | 0.0307 | 0.0214 | 0.0050 | 0.0004 |
| 5 | 1.0000 | 1.0000 | 0.9998 | 0.9996 | 0.9988 | 0.9958 | 0.9887 | 0.9803 | 0.9747 | 0.9502 | 0.9115 | 0.8555 | 0.7799 | 0.6846 | 0.5722 | 0.5318 | 0.4482 | 0.3215 | 0.2031 | 0.1348 | 0.1052 | 0.0381 | 0.0058 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9987 | 0.9974 | 0.9964 | 0.9915 | 0.9819 | 0.9648 | 0.9368 | 0.8936 | 0.8309 | 0.8049 | 0.7447 | 0.6329 | 0.4967 | 0.3953 | 0.3428 | 0.1869 | 0.0572 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9998 | 0.9993 | 0.9983 | 0.9961 | 0.9916 | 0.9832 | 0.9681 | 0.9610 | 0.9424 | 0.8999 | 0.8322 | 0.7674 | 0.7275 | 0.5695 | 0.3366 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

## CUMULATIVE BINOMIAL PROBABILITIES

| $n = 9$, $p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.6302 | 0.3874 | 0.2316 | 0.1938 | 0.1342 | 0.0751 | 0.0404 | 0.0260 | 0.0207 | 0.0101 | 0.0046 | 0.0020 | 0.0008 | 0.0003 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.9288 | 0.7748 | 0.5995 | 0.5427 | 0.4362 | 0.3003 | 0.1960 | 0.1431 | 0.1211 | 0.0705 | 0.0385 | 0.0195 | 0.0091 | 0.0038 | 0.0014 | 0.0010 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9916 | 0.9470 | 0.8591 | 0.8217 | 0.7382 | 0.6007 | 0.4628 | 0.3772 | 0.3373 | 0.2318 | 0.1495 | 0.0898 | 0.0498 | 0.0250 | 0.0112 | 0.0083 | 0.0043 | 0.0013 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9994 | 0.9917 | 0.9661 | 0.9520 | 0.9144 | 0.8343 | 0.7297 | 0.6503 | 0.6089 | 0.4826 | 0.3614 | 0.2539 | 0.1658 | 0.0994 | 0.0536 | 0.0424 | 0.0253 | 0.0100 | 0.0031 | 0.0011 | 0.0006 | 0.0001 | 0.0000 |
| 4 | 1.0000 | 0.9991 | 0.9944 | 0.9910 | 0.9804 | 0.9511 | 0.9012 | 0.8552 | 0.8283 | 0.7334 | 0.6214 | 0.5000 | 0.3786 | 0.2666 | 0.1717 | 0.1448 | 0.0988 | 0.0489 | 0.0196 | 0.0090 | 0.0056 | 0.0009 | 0.0000 |
| 5 | 1.0000 | 0.9999 | 0.9994 | 0.9989 | 0.9969 | 0.9900 | 0.9747 | 0.9576 | 0.9464 | 0.9006 | 0.8342 | 0.7461 | 0.6386 | 0.5174 | 0.3911 | 0.3497 | 0.2703 | 0.1657 | 0.0856 | 0.0480 | 0.0339 | 0.0083 | 0.0006 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9987 | 0.9957 | 0.9917 | 0.9888 | 0.9750 | 0.9502 | 0.9102 | 0.8505 | 0.7682 | 0.6627 | 0.6228 | 0.5372 | 0.3993 | 0.2618 | 0.1783 | 0.1409 | 0.0530 | 0.0084 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9990 | 0.9986 | 0.9962 | 0.9909 | 0.9805 | 0.9615 | 0.9295 | 0.8789 | 0.8569 | 0.8040 | 0.6997 | 0.5638 | 0.4573 | 0.4005 | 0.2252 | 0.0712 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9992 | 0.9980 | 0.9954 | 0.9899 | 0.9793 | 0.9740 | 0.9596 | 0.9249 | 0.8658 | 0.8062 | 0.7684 | 0.6126 | 0.3698 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $n = 10$, $p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.5987 | 0.3487 | 0.1969 | 0.1615 | 0.1074 | 0.0563 | 0.0282 | 0.0173 | 0.0135 | 0.0060 | 0.0025 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.9139 | 0.7361 | 0.5443 | 0.4845 | 0.3758 | 0.2440 | 0.1493 | 0.1040 | 0.0860 | 0.0464 | 0.0233 | 0.0107 | 0.0045 | 0.0017 | 0.0005 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9885 | 0.9298 | 0.8202 | 0.7752 | 0.6778 | 0.5256 | 0.3828 | 0.2991 | 0.2616 | 0.1673 | 0.0996 | 0.0547 | 0.0274 | 0.0123 | 0.0048 | 0.0034 | 0.0016 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9990 | 0.9872 | 0.9500 | 0.9303 | 0.8791 | 0.7759 | 0.6496 | 0.5593 | 0.5138 | 0.3823 | 0.2660 | 0.1719 | 0.1020 | 0.0548 | 0.0260 | 0.0197 | 0.0106 | 0.0035 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 4 | 0.9999 | 0.9984 | 0.9901 | 0.9845 | 0.9672 | 0.9219 | 0.8497 | 0.7869 | 0.7515 | 0.6331 | 0.5044 | 0.3770 | 0.2616 | 0.1662 | 0.0949 | 0.0766 | 0.0473 | 0.0197 | 0.0064 | 0.0024 | 0.0014 | 0.0001 | 0.0000 |
| 5 | 1.0000 | 0.9999 | 0.9986 | 0.9976 | 0.9936 | 0.9803 | 0.9527 | 0.9234 | 0.9051 | 0.8338 | 0.7384 | 0.6230 | 0.4956 | 0.3669 | 0.2485 | 0.2131 | 0.1503 | 0.0781 | 0.0328 | 0.0155 | 0.0099 | 0.0016 | 0.0001 |
| 6 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 | 0.9965 | 0.9894 | 0.9803 | 0.9740 | 0.9452 | 0.8980 | 0.8281 | 0.7340 | 0.6177 | 0.4862 | 0.4407 | 0.3504 | 0.2241 | 0.1209 | 0.0697 | 0.0500 | 0.0128 | 0.0010 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9984 | 0.9966 | 0.9952 | 0.9877 | 0.9726 | 0.9453 | 0.9004 | 0.8327 | 0.7384 | 0.7009 | 0.6172 | 0.4744 | 0.3222 | 0.2248 | 0.1798 | 0.0702 | 0.0115 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9995 | 0.9983 | 0.9955 | 0.9893 | 0.9767 | 0.9536 | 0.9140 | 0.8960 | 0.8507 | 0.7560 | 0.6242 | 0.5155 | 0.4557 | 0.2639 | 0.0861 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9975 | 0.9940 | 0.9865 | 0.9827 | 0.9718 | 0.9437 | 0.8926 | 0.8385 | 0.8031 | 0.6513 | 0.4013 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $n = 12$, $p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.5404 | 0.2824 | 0.1422 | 0.1122 | 0.0687 | 0.0317 | 0.0138 | 0.0077 | 0.0057 | 0.0022 | 0.0008 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.8816 | 0.6590 | 0.4435 | 0.3813 | 0.2749 | 0.1584 | 0.0850 | 0.0540 | 0.0424 | 0.0196 | 0.0083 | 0.0032 | 0.0011 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9804 | 0.8891 | 0.7358 | 0.6774 | 0.5583 | 0.3907 | 0.2528 | 0.1811 | 0.1513 | 0.0834 | 0.0421 | 0.0193 | 0.0079 | 0.0028 | 0.0008 | 0.0005 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9978 | 0.9744 | 0.9078 | 0.8748 | 0.7946 | 0.6488 | 0.4925 | 0.3931 | 0.3467 | 0.2253 | 0.1345 | 0.0730 | 0.0356 | 0.0153 | 0.0056 | 0.0039 | 0.0017 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9998 | 0.9957 | 0.9761 | 0.9636 | 0.9274 | 0.8424 | 0.7237 | 0.6315 | 0.5833 | 0.4382 | 0.3044 | 0.1938 | 0.1117 | 0.0573 | 0.0255 | 0.0188 | 0.0095 | 0.0028 | 0.0006 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 5 | 1.0000 | 0.9995 | 0.9954 | 0.9921 | 0.9806 | 0.9456 | 0.8822 | 0.8223 | 0.7873 | 0.6652 | 0.5269 | 0.3872 | 0.2607 | 0.1582 | 0.0846 | 0.0664 | 0.0386 | 0.0143 | 0.0039 | 0.0013 | 0.0007 | 0.0001 | 0.0000 |
| 6 | 1.0000 | 0.9999 | 0.9993 | 0.9987 | 0.9961 | 0.9857 | 0.9614 | 0.9336 | 0.9154 | 0.8418 | 0.7393 | 0.6128 | 0.4731 | 0.3348 | 0.2127 | 0.1777 | 0.1178 | 0.0544 | 0.0194 | 0.0079 | 0.0046 | 0.0005 | 0.0000 |
| 7 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9972 | 0.9905 | 0.9812 | 0.9745 | 0.9427 | 0.8883 | 0.8062 | 0.6956 | 0.5618 | 0.4167 | 0.3685 | 0.2763 | 0.1576 | 0.0726 | 0.0364 | 0.0239 | 0.0043 | 0.0002 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9983 | 0.9961 | 0.9944 | 0.9847 | 0.9644 | 0.9270 | 0.8655 | 0.7747 | 0.6533 | 0.6069 | 0.5075 | 0.3512 | 0.2054 | 0.1252 | 0.0922 | 0.0256 | 0.0022 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9995 | 0.9992 | 0.9972 | 0.9921 | 0.9807 | 0.9579 | 0.9166 | 0.8487 | 0.8189 | 0.7472 | 0.6093 | 0.4417 | 0.3226 | 0.2642 | 0.1109 | 0.0196 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9989 | 0.9968 | 0.9917 | 0.9804 | 0.9576 | 0.9460 | 0.9150 | 0.8416 | 0.7251 | 0.6187 | 0.5565 | 0.3410 | 0.1184 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9992 | 0.9978 | 0.9943 | 0.9923 | 0.9862 | 0.9683 | 0.9313 | 0.8878 | 0.8578 | 0.7176 | 0.4596 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

# CUMULATIVE BINOMIAL PROBABILITIES

| $n = 14$<br>$p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.4877 | 0.2288 | 0.1028 | 0.0779 | 0.0440 | 0.0178 | 0.0068 | 0.0034 | 0.0024 | 0.0008 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.8470 | 0.5846 | 0.3567 | 0.2960 | 0.1979 | 0.1010 | 0.0475 | 0.0274 | 0.0205 | 0.0081 | 0.0029 | 0.0009 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9699 | 0.8416 | 0.6479 | 0.5795 | 0.4481 | 0.2811 | 0.1608 | 0.1053 | 0.0839 | 0.0398 | 0.0170 | 0.0065 | 0.0022 | 0.0006 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9958 | 0.9559 | 0.8535 | 0.8063 | 0.6982 | 0.5213 | 0.3552 | 0.2612 | 0.2205 | 0.1243 | 0.0632 | 0.0287 | 0.0114 | 0.0039 | 0.0011 | 0.0007 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9996 | 0.9908 | 0.9533 | 0.9310 | 0.8702 | 0.7415 | 0.5842 | 0.4755 | 0.4227 | 0.2793 | 0.1672 | 0.0898 | 0.0426 | 0.0175 | 0.0060 | 0.0040 | 0.0017 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 1.0000 | 0.9985 | 0.9885 | 0.9809 | 0.9561 | 0.8883 | 0.7805 | 0.6898 | 0.6405 | 0.4859 | 0.3373 | 0.2120 | 0.1189 | 0.0583 | 0.0243 | 0.0174 | 0.0083 | 0.0022 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 1.0000 | 0.9998 | 0.9978 | 0.9959 | 0.9884 | 0.9617 | 0.9067 | 0.8505 | 0.8164 | 0.6925 | 0.5461 | 0.3953 | 0.2586 | 0.1501 | 0.0753 | 0.0576 | 0.0315 | 0.0103 | 0.0024 | 0.0007 | 0.0003 | 0.0000 | 0.0000 |
| 7 | 1.0000 | 1.0000 | 0.9997 | 0.9993 | 0.9976 | 0.9897 | 0.9685 | 0.9424 | 0.9247 | 0.8499 | 0.7414 | 0.6047 | 0.4539 | 0.3075 | 0.1836 | 0.1495 | 0.0933 | 0.0383 | 0.0116 | 0.0041 | 0.0022 | 0.0002 | 0.0000 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9978 | 0.9917 | 0.9826 | 0.9757 | 0.9417 | 0.8811 | 0.7880 | 0.6627 | 0.5141 | 0.3595 | 0.3102 | 0.2195 | 0.1117 | 0.0439 | 0.0191 | 0.0115 | 0.0015 | 0.0000 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9983 | 0.9960 | 0.9940 | 0.9825 | 0.9574 | 0.9102 | 0.8328 | 0.7207 | 0.5773 | 0.5245 | 0.4158 | 0.2585 | 0.1298 | 0.0690 | 0.0467 | 0.0092 | 0.0004 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9993 | 0.9989 | 0.9961 | 0.9886 | 0.9713 | 0.9368 | 0.8757 | 0.7795 | 0.7388 | 0.6448 | 0.4787 | 0.3018 | 0.1937 | 0.1465 | 0.0441 | 0.0042 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9994 | 0.9978 | 0.9935 | 0.9830 | 0.9602 | 0.9161 | 0.8947 | 0.8392 | 0.7189 | 0.5519 | 0.4205 | 0.3521 | 0.1584 | 0.0301 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 | 0.9971 | 0.9919 | 0.9795 | 0.9726 | 0.9525 | 0.8990 | 0.8021 | 0.7040 | 0.6433 | 0.4154 | 0.1530 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9992 | 0.9976 | 0.9966 | 0.9932 | 0.9822 | 0.9560 | 0.9221 | 0.8972 | 0.7712 | 0.5123 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $n = 16$<br>$p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.4401 | 0.1853 | 0.0743 | 0.0541 | 0.0281 | 0.0100 | 0.0033 | 0.0015 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.8108 | 0.5147 | 0.2839 | 0.2272 | 0.1407 | 0.0635 | 0.0261 | 0.0137 | 0.0098 | 0.0033 | 0.0010 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9571 | 0.7892 | 0.5614 | 0.4868 | 0.3518 | 0.1971 | 0.0994 | 0.0594 | 0.0451 | 0.0183 | 0.0066 | 0.0021 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9930 | 0.9316 | 0.7899 | 0.7291 | 0.5981 | 0.4050 | 0.2459 | 0.1659 | 0.1339 | 0.0651 | 0.0281 | 0.0106 | 0.0035 | 0.0009 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9991 | 0.9830 | 0.9209 | 0.8866 | 0.7982 | 0.6302 | 0.4499 | 0.3391 | 0.2892 | 0.1666 | 0.0853 | 0.0384 | 0.0149 | 0.0049 | 0.0013 | 0.0008 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.9999 | 0.9967 | 0.9765 | 0.9622 | 0.9183 | 0.8103 | 0.6598 | 0.5469 | 0.4900 | 0.3288 | 0.1976 | 0.1051 | 0.0486 | 0.0191 | 0.0062 | 0.0040 | 0.0016 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 1.0000 | 0.9995 | 0.9944 | 0.9899 | 0.9733 | 0.9204 | 0.8247 | 0.7374 | 0.6881 | 0.5272 | 0.3660 | 0.2272 | 0.1241 | 0.0583 | 0.0229 | 0.0159 | 0.0071 | 0.0016 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 1.0000 | 0.9999 | 0.9989 | 0.9979 | 0.9930 | 0.9729 | 0.9256 | 0.8735 | 0.8406 | 0.7161 | 0.5629 | 0.4018 | 0.2559 | 0.1423 | 0.0671 | 0.0500 | 0.0257 | 0.0075 | 0.0015 | 0.0004 | 0.0002 | 0.0000 | 0.0000 |
| 8 | 1.0000 | 1.0000 | 0.9998 | 0.9996 | 0.9985 | 0.9925 | 0.9743 | 0.9500 | 0.9329 | 0.8577 | 0.7441 | 0.5982 | 0.4371 | 0.2839 | 0.1594 | 0.1265 | 0.0744 | 0.0271 | 0.0070 | 0.0021 | 0.0011 | 0.0001 | 0.0000 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9984 | 0.9929 | 0.9841 | 0.9771 | 0.9417 | 0.8759 | 0.7728 | 0.6340 | 0.4728 | 0.3119 | 0.2626 | 0.1753 | 0.0796 | 0.0267 | 0.0101 | 0.0056 | 0.0005 | 0.0000 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9960 | 0.9938 | 0.9809 | 0.9514 | 0.8949 | 0.8024 | 0.6712 | 0.5100 | 0.4531 | 0.3402 | 0.1897 | 0.0817 | 0.0378 | 0.0235 | 0.0033 | 0.0001 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9992 | 0.9987 | 0.9951 | 0.9851 | 0.9616 | 0.9147 | 0.8334 | 0.7108 | 0.6609 | 0.5501 | 0.3698 | 0.2018 | 0.1134 | 0.0791 | 0.0170 | 0.0009 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9991 | 0.9965 | 0.9894 | 0.9719 | 0.9349 | 0.8661 | 0.8341 | 0.7541 | 0.5950 | 0.4019 | 0.2709 | 0.2101 | 0.0684 | 0.0070 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9979 | 0.9934 | 0.9817 | 0.9549 | 0.9406 | 0.9006 | 0.8029 | 0.6482 | 0.5132 | 0.4386 | 0.2108 | 0.0429 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9967 | 0.9902 | 0.9863 | 0.9739 | 0.9365 | 0.8593 | 0.7728 | 0.7161 | 0.4853 | 0.1892 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9985 | 0.9967 | 0.9900 | 0.9719 | 0.9459 | 0.9257 | 0.8147 | 0.5599 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

# CUMULATIVE BINOMIAL PROBABILITIES

| $n = 18$ $p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.3972 | 0.1501 | 0.0536 | 0.0376 | 0.0180 | 0.0056 | 0.0016 | 0.0007 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.7735 | 0.4503 | 0.2241 | 0.1728 | 0.0991 | 0.0395 | 0.0142 | 0.0068 | 0.0046 | 0.0013 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9419 | 0.7338 | 0.4797 | 0.4027 | 0.2713 | 0.1353 | 0.0600 | 0.0326 | 0.0236 | 0.0082 | 0.0025 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9891 | 0.9018 | 0.7202 | 0.6479 | 0.5010 | 0.3057 | 0.1646 | 0.1017 | 0.0783 | 0.0328 | 0.0120 | 0.0038 | 0.0010 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9985 | 0.9718 | 0.8794 | 0.8318 | 0.7164 | 0.5187 | 0.3327 | 0.2311 | 0.1886 | 0.0942 | 0.0411 | 0.0154 | 0.0049 | 0.0013 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.9998 | 0.9936 | 0.9581 | 0.9347 | 0.8671 | 0.7175 | 0.5344 | 0.4122 | 0.3550 | 0.2088 | 0.1077 | 0.0481 | 0.0183 | 0.0058 | 0.0014 | 0.0009 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 1.0000 | 0.9988 | 0.9882 | 0.9794 | 0.9487 | 0.8610 | 0.7217 | 0.6085 | 0.5491 | 0.3743 | 0.2258 | 0.1189 | 0.0537 | 0.0203 | 0.0062 | 0.0039 | 0.0014 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 1.0000 | 0.9998 | 0.9973 | 0.9947 | 0.9837 | 0.9431 | 0.8593 | 0.7767 | 0.7283 | 0.5634 | 0.3915 | 0.2403 | 0.1280 | 0.0576 | 0.0212 | 0.0144 | 0.0061 | 0.0012 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 1.0000 | 1.0000 | 0.9995 | 0.9989 | 0.9957 | 0.9807 | 0.9404 | 0.8924 | 0.8609 | 0.7368 | 0.5778 | 0.4073 | 0.2527 | 0.1347 | 0.0597 | 0.0433 | 0.0210 | 0.0054 | 0.0009 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| 9 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9991 | 0.9946 | 0.9790 | 0.9567 | 0.9403 | 0.8653 | 0.7473 | 0.5927 | 0.4222 | 0.2632 | 0.1391 | 0.1076 | 0.0596 | 0.0193 | 0.0043 | 0.0011 | 0.0005 | 0.0000 | 0.0000 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9939 | 0.9856 | 0.9788 | 0.9424 | 0.8720 | 0.7597 | 0.6085 | 0.4366 | 0.2717 | 0.2233 | 0.1407 | 0.0569 | 0.0163 | 0.0053 | 0.0027 | 0.0002 | 0.0000 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9986 | 0.9961 | 0.9938 | 0.9797 | 0.9463 | 0.8811 | 0.7742 | 0.6257 | 0.4509 | 0.3915 | 0.2783 | 0.1390 | 0.0513 | 0.0206 | 0.0118 | 0.0012 | 0.0000 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9991 | 0.9986 | 0.9942 | 0.9817 | 0.9519 | 0.8923 | 0.7912 | 0.6450 | 0.5878 | 0.4656 | 0.2825 | 0.1329 | 0.0653 | 0.0419 | 0.0064 | 0.0002 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9987 | 0.9951 | 0.9846 | 0.9589 | 0.9058 | 0.8114 | 0.7689 | 0.6673 | 0.4813 | 0.2836 | 0.1682 | 0.1206 | 0.0282 | 0.0015 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9990 | 0.9962 | 0.9880 | 0.9672 | 0.9217 | 0.8983 | 0.8354 | 0.6943 | 0.4990 | 0.3521 | 0.2798 | 0.0982 | 0.0109 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9975 | 0.9918 | 0.9764 | 0.9674 | 0.9400 | 0.8647 | 0.7287 | 0.5973 | 0.5203 | 0.2662 | 0.0581 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9987 | 0.9954 | 0.9932 | 0.9858 | 0.9605 | 0.9009 | 0.8272 | 0.7759 | 0.5497 | 0.2265 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9993 | 0.9984 | 0.9944 | 0.9820 | 0.9624 | 0.9464 | 0.8499 | 0.6028 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $n = 20$ $p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.3585 | 0.1216 | 0.0388 | 0.0261 | 0.0115 | 0.0032 | 0.0008 | 0.0003 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.7358 | 0.3917 | 0.1756 | 0.1304 | 0.0692 | 0.0243 | 0.0076 | 0.0033 | 0.0021 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.9245 | 0.6769 | 0.4049 | 0.3287 | 0.2061 | 0.0913 | 0.0355 | 0.0176 | 0.0121 | 0.0036 | 0.0009 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9841 | 0.8670 | 0.6477 | 0.5665 | 0.4114 | 0.2252 | 0.1071 | 0.0604 | 0.0444 | 0.0160 | 0.0049 | 0.0013 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9974 | 0.9568 | 0.8298 | 0.7687 | 0.6296 | 0.4148 | 0.2375 | 0.1515 | 0.1182 | 0.0510 | 0.0189 | 0.0059 | 0.0015 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.9997 | 0.9887 | 0.9327 | 0.8982 | 0.8042 | 0.6172 | 0.4164 | 0.2972 | 0.2454 | 0.1256 | 0.0553 | 0.0207 | 0.0064 | 0.0016 | 0.0003 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 1.0000 | 0.9976 | 0.9781 | 0.9629 | 0.9133 | 0.7858 | 0.6080 | 0.4793 | 0.4166 | 0.2500 | 0.1299 | 0.0577 | 0.0214 | 0.0065 | 0.0015 | 0.0009 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 1.0000 | 0.9996 | 0.9941 | 0.9887 | 0.9679 | 0.8982 | 0.7723 | 0.6615 | 0.6010 | 0.4159 | 0.2520 | 0.1316 | 0.0580 | 0.0210 | 0.0060 | 0.0037 | 0.0013 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 1.0000 | 0.9999 | 0.9987 | 0.9972 | 0.9900 | 0.9591 | 0.8867 | 0.8095 | 0.7624 | 0.5956 | 0.4143 | 0.2517 | 0.1308 | 0.0565 | 0.0196 | 0.0130 | 0.0051 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 1.0000 | 1.0000 | 0.9998 | 0.9994 | 0.9974 | 0.9861 | 0.9520 | 0.9081 | 0.8782 | 0.7553 | 0.5914 | 0.4119 | 0.2493 | 0.1275 | 0.0532 | 0.0376 | 0.0171 | 0.0039 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9961 | 0.9829 | 0.9624 | 0.9468 | 0.8725 | 0.7507 | 0.5881 | 0.4086 | 0.2447 | 0.1218 | 0.0919 | 0.0480 | 0.0139 | 0.0026 | 0.0006 | 0.0002 | 0.0000 | 0.0000 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9949 | 0.9870 | 0.9804 | 0.9435 | 0.8692 | 0.7483 | 0.5857 | 0.4044 | 0.2376 | 0.1905 | 0.1133 | 0.0409 | 0.0100 | 0.0028 | 0.0013 | 0.0001 | 0.0000 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9987 | 0.9963 | 0.9940 | 0.9790 | 0.9420 | 0.8684 | 0.7480 | 0.5841 | 0.3990 | 0.3385 | 0.2277 | 0.1018 | 0.0321 | 0.0113 | 0.0059 | 0.0004 | 0.0000 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9991 | 0.9985 | 0.9935 | 0.9786 | 0.9423 | 0.8701 | 0.7500 | 0.5834 | 0.5207 | 0.3920 | 0.2142 | 0.0867 | 0.0371 | 0.0219 | 0.0024 | 0.0000 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9997 | 0.9984 | 0.9936 | 0.9793 | 0.9447 | 0.8744 | 0.7546 | 0.7028 | 0.5836 | 0.3828 | 0.1958 | 0.1018 | 0.0673 | 0.0113 | 0.0003 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9985 | 0.9941 | 0.9811 | 0.9490 | 0.8818 | 0.8485 | 0.7625 | 0.5852 | 0.3704 | 0.2313 | 0.1702 | 0.0432 | 0.0026 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9987 | 0.9951 | 0.9840 | 0.9556 | 0.9396 | 0.8929 | 0.7748 | 0.5886 | 0.4335 | 0.3523 | 0.1330 | 0.0159 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9991 | 0.9964 | 0.9879 | 0.9824 | 0.9645 | 0.9087 | 0.7939 | 0.6713 | 0.5951 | 0.3231 | 0.0755 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9979 | 0.9967 | 0.9924 | 0.9757 | 0.9308 | 0.8696 | 0.8244 | 0.6083 | 0.2642 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9997 | 0.9992 | 0.9968 | 0.9885 | 0.9739 | 0.9612 | 0.8784 | 0.6415 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

# CUMULATIVE BINOMIAL PROBABILITIES

| $n = 25$ $p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.2774 | 0.0718 | 0.0172 | 0.0105 | 0.0038 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.6424 | 0.2712 | 0.0931 | 0.0629 | 0.0274 | 0.0070 | 0.0016 | 0.0005 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.8729 | 0.5371 | 0.2537 | 0.1887 | 0.0982 | 0.0321 | 0.0090 | 0.0035 | 0.0021 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9659 | 0.7636 | 0.4711 | 0.3816 | 0.2340 | 0.0962 | 0.0332 | 0.0149 | 0.0097 | 0.0024 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9928 | 0.9020 | 0.6821 | 0.5937 | 0.4207 | 0.2137 | 0.0905 | 0.0462 | 0.0320 | 0.0095 | 0.0023 | 0.0005 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.9988 | 0.9666 | 0.8385 | 0.7720 | 0.6167 | 0.3783 | 0.1935 | 0.1120 | 0.0826 | 0.0294 | 0.0086 | 0.0020 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.9998 | 0.9905 | 0.9305 | 0.8908 | 0.7800 | 0.5611 | 0.3407 | 0.2215 | 0.1734 | 0.0736 | 0.0258 | 0.0073 | 0.0016 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 1.0000 | 0.9977 | 0.9745 | 0.9553 | 0.8909 | 0.7265 | 0.5118 | 0.3703 | 0.3061 | 0.1536 | 0.0639 | 0.0216 | 0.0058 | 0.0012 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 1.0000 | 0.9995 | 0.9920 | 0.9843 | 0.9532 | 0.8506 | 0.6769 | 0.5376 | 0.4668 | 0.2735 | 0.1340 | 0.0539 | 0.0174 | 0.0043 | 0.0008 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 1.0000 | 0.9999 | 0.9979 | 0.9953 | 0.9827 | 0.9287 | 0.8106 | 0.6956 | 0.6303 | 0.4246 | 0.2424 | 0.1148 | 0.0440 | 0.0132 | 0.0029 | 0.0016 | 0.0005 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 1.0000 | 1.0000 | 0.9995 | 0.9988 | 0.9944 | 0.9703 | 0.9022 | 0.8220 | 0.7712 | 0.5858 | 0.3843 | 0.2122 | 0.0960 | 0.0344 | 0.0093 | 0.0056 | 0.0018 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 11 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9985 | 0.9893 | 0.9558 | 0.9082 | 0.8746 | 0.7323 | 0.5426 | 0.3450 | 0.1827 | 0.0778 | 0.0255 | 0.0164 | 0.0060 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9966 | 0.9825 | 0.9585 | 0.9396 | 0.8462 | 0.6937 | 0.5000 | 0.3063 | 0.1538 | 0.0604 | 0.0415 | 0.0175 | 0.0034 | 0.0004 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9940 | 0.9836 | 0.9745 | 0.9222 | 0.8173 | 0.6550 | 0.4574 | 0.2677 | 0.1254 | 0.0918 | 0.0442 | 0.0107 | 0.0015 | 0.0003 | 0.0001 | 0.0000 | 0.0000 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9982 | 0.9944 | 0.9907 | 0.9656 | 0.9040 | 0.7878 | 0.6157 | 0.4142 | 0.2288 | 0.1780 | 0.0978 | 0.0297 | 0.0056 | 0.0012 | 0.0005 | 0.0000 | 0.0000 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9995 | 0.9984 | 0.9971 | 0.9868 | 0.9560 | 0.8852 | 0.7576 | 0.5754 | 0.3697 | 0.3044 | 0.1894 | 0.0713 | 0.0173 | 0.0047 | 0.0021 | 0.0001 | 0.0000 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9992 | 0.9957 | 0.9826 | 0.9461 | 0.8660 | 0.7265 | 0.5332 | 0.4624 | 0.3231 | 0.1494 | 0.0468 | 0.0157 | 0.0080 | 0.0005 | 0.0000 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9988 | 0.9942 | 0.9784 | 0.9361 | 0.8464 | 0.6939 | 0.6297 | 0.4882 | 0.2735 | 0.1091 | 0.0447 | 0.0255 | 0.0023 | 0.0000 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9984 | 0.9927 | 0.9742 | 0.9264 | 0.8266 | 0.7785 | 0.6593 | 0.4389 | 0.2200 | 0.1092 | 0.0695 | 0.0095 | 0.0002 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9980 | 0.9914 | 0.9706 | 0.9174 | 0.8880 | 0.8065 | 0.6217 | 0.3833 | 0.2280 | 0.1615 | 0.0334 | 0.0012 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9977 | 0.9905 | 0.9680 | 0.9538 | 0.9095 | 0.7863 | 0.5793 | 0.4063 | 0.3179 | 0.0980 | 0.0072 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9976 | 0.9903 | 0.9851 | 0.9668 | 0.9038 | 0.7660 | 0.6184 | 0.5289 | 0.2364 | 0.0341 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9979 | 0.9965 | 0.9910 | 0.9679 | 0.9018 | 0.8113 | 0.7463 | 0.4629 | 0.1271 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 | 0.9984 | 0.9930 | 0.9726 | 0.9371 | 0.9069 | 0.7288 | 0.3576 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9962 | 0.9895 | 0.9828 | 0.9282 | 0.7226 |
| 25 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

# CUMULATIVE BINOMIAL PROBABILITIES

| $n = 30$ <br> $p$ | 0.05 | 0.1 | 0.15 | 1/6 | 0.2 | 0.25 | 0.3 | 1/3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 2/3 | 0.7 | 0.75 | 0.8 | 5/6 | 0.85 | 0.9 | 0.95 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.2146 | 0.0424 | 0.0076 | 0.0042 | 0.0012 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.5535 | 0.1837 | 0.0480 | 0.0295 | 0.0105 | 0.0020 | 0.0003 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.8122 | 0.4114 | 0.1514 | 0.1028 | 0.0442 | 0.0106 | 0.0021 | 0.0007 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.9392 | 0.6474 | 0.3217 | 0.2396 | 0.1227 | 0.0374 | 0.0093 | 0.0033 | 0.0019 | 0.0003 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.9844 | 0.8245 | 0.5245 | 0.4243 | 0.2552 | 0.0979 | 0.0302 | 0.0122 | 0.0075 | 0.0015 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 0.9967 | 0.9268 | 0.7106 | 0.6164 | 0.4275 | 0.2026 | 0.0766 | 0.0355 | 0.0233 | 0.0057 | 0.0011 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 6 | 0.9994 | 0.9742 | 0.8474 | 0.7765 | 0.6070 | 0.3481 | 0.1595 | 0.0838 | 0.0586 | 0.0172 | 0.0040 | 0.0007 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 0.9999 | 0.9922 | 0.9302 | 0.8863 | 0.7608 | 0.5143 | 0.2814 | 0.1668 | 0.1238 | 0.0435 | 0.0121 | 0.0026 | 0.0004 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 8 | 1.0000 | 0.9980 | 0.9722 | 0.9494 | 0.8713 | 0.6736 | 0.4315 | 0.2860 | 0.2247 | 0.0940 | 0.0312 | 0.0081 | 0.0016 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 1.0000 | 0.9995 | 0.9903 | 0.9803 | 0.9389 | 0.8034 | 0.5888 | 0.4317 | 0.3575 | 0.1763 | 0.0694 | 0.0214 | 0.0050 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 1.0000 | 0.9999 | 0.9971 | 0.9933 | 0.9744 | 0.8943 | 0.7304 | 0.5848 | 0.5078 | 0.2915 | 0.1350 | 0.0494 | 0.0138 | 0.0029 | 0.0004 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 11 | 1.0000 | 1.0000 | 0.9992 | 0.9980 | 0.9905 | 0.9493 | 0.8407 | 0.7239 | 0.6548 | 0.4311 | 0.2327 | 0.1002 | 0.0334 | 0.0083 | 0.0014 | 0.0007 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 12 | 1.0000 | 1.0000 | 0.9998 | 0.9995 | 0.9969 | 0.9784 | 0.9155 | 0.8340 | 0.7802 | 0.5785 | 0.3592 | 0.1808 | 0.0714 | 0.0212 | 0.0045 | 0.0025 | 0.0006 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9918 | 0.9599 | 0.9102 | 0.8737 | 0.7145 | 0.5025 | 0.2923 | 0.1356 | 0.0481 | 0.0124 | 0.0072 | 0.0021 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9973 | 0.9831 | 0.9565 | 0.9348 | 0.8246 | 0.6448 | 0.4278 | 0.2309 | 0.0971 | 0.0301 | 0.0188 | 0.0064 | 0.0008 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9992 | 0.9936 | 0.9812 | 0.9699 | 0.9029 | 0.7691 | 0.5722 | 0.3552 | 0.1754 | 0.0652 | 0.0435 | 0.0169 | 0.0027 | 0.0002 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9979 | 0.9928 | 0.9876 | 0.9519 | 0.8644 | 0.7077 | 0.4975 | 0.2855 | 0.1263 | 0.0898 | 0.0401 | 0.0082 | 0.0009 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 17 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9994 | 0.9975 | 0.9955 | 0.9788 | 0.9286 | 0.8192 | 0.6408 | 0.4215 | 0.2198 | 0.1660 | 0.0845 | 0.0216 | 0.0031 | 0.0005 | 0.0002 | 0.0000 | 0.0000 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9993 | 0.9986 | 0.9917 | 0.9666 | 0.8998 | 0.7673 | 0.5689 | 0.3452 | 0.2761 | 0.1593 | 0.0507 | 0.0095 | 0.0020 | 0.0008 | 0.0000 | 0.0000 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9996 | 0.9971 | 0.9862 | 0.9506 | 0.8650 | 0.7085 | 0.4922 | 0.4152 | 0.2696 | 0.1057 | 0.0256 | 0.0067 | 0.0029 | 0.0001 | 0.0000 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9991 | 0.9950 | 0.9786 | 0.9306 | 0.8237 | 0.6425 | 0.5683 | 0.4112 | 0.1966 | 0.0611 | 0.0197 | 0.0097 | 0.0005 | 0.0000 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9984 | 0.9919 | 0.9688 | 0.9060 | 0.7753 | 0.7140 | 0.5685 | 0.3264 | 0.1287 | 0.0506 | 0.0278 | 0.0020 | 0.0000 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9996 | 0.9974 | 0.9879 | 0.9565 | 0.8762 | 0.8332 | 0.7186 | 0.4857 | 0.2392 | 0.1137 | 0.0698 | 0.0078 | 0.0001 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9960 | 0.9828 | 0.9414 | 0.9162 | 0.8405 | 0.6519 | 0.3930 | 0.2235 | 0.1526 | 0.0258 | 0.0006 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9989 | 0.9943 | 0.9767 | 0.9645 | 0.9234 | 0.7974 | 0.5725 | 0.3836 | 0.2894 | 0.0732 | 0.0033 |
| 25 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9985 | 0.9925 | 0.9878 | 0.9698 | 0.9021 | 0.7448 | 0.5757 | 0.4755 | 0.1755 | 0.0156 |
| 26 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9981 | 0.9967 | 0.9907 | 0.9626 | 0.8773 | 0.7604 | 0.6783 | 0.3526 | 0.0608 |
| 27 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9997 | 0.9993 | 0.9979 | 0.9894 | 0.9558 | 0.8972 | 0.8486 | 0.5886 | 0.1878 |
| 28 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9980 | 0.9895 | 0.9705 | 0.9520 | 0.8163 | 0.4465 |
| 29 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9998 | 0.9988 | 0.9958 | 0.9924 | 0.9576 | 0.7854 |
| 30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

## CUMULATIVE POISSON PROBABILITIES

$$\mathrm{P}(X \leqslant x) = \sum_{r=0}^{x} \mathrm{e}^{-\lambda} \frac{\lambda^r}{r!}$$

| λ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.9900 | 0.9802 | 0.9704 | 0.9608 | 0.9512 | 0.9418 | 0.9324 | 0.9231 | 0.9139 |
| 1 | 1.0000 | 0.9998 | 0.9996 | 0.9992 | 0.9988 | 0.9983 | 0.9977 | 0.9970 | 0.9962 |
| 2 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 |
| 3 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 |
|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 |
| 1 | 0.9953 | 0.9825 | 0.9631 | 0.9384 | 0.9098 | 0.8781 | 0.8442 | 0.8088 | 0.7725 |
| 2 | 0.9998 | 0.9989 | 0.9964 | 0.9921 | 0.9856 | 0.9769 | 0.9659 | 0.9526 | 0.9371 |
| 3 | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9982 | 0.9966 | 0.9942 | 0.9909 | 0.9865 |
| 4 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9986 | 0.9977 |
| 5 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9997 |
| 6 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.3679 | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 | 0.1496 |
| 1 | 0.7358 | 0.6990 | 0.6626 | 0.6268 | 0.5918 | 0.5578 | 0.5249 | 0.4932 | 0.4628 | 0.4337 |
| 2 | 0.9197 | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 | 0.7834 | 0.7572 | 0.7306 | 0.7037 |
| 3 | 0.9810 | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 | 0.9212 | 0.9068 | 0.8913 | 0.8747 |
| 4 | 0.9963 | 0.9946 | 0.9923 | 0.9893 | 0.9857 | 0.9814 | 0.9763 | 0.9704 | 0.9636 | 0.9559 |
| 5 | 0.9994 | 0.9990 | 0.9985 | 0.9978 | 0.9968 | 0.9955 | 0.9940 | 0.9920 | 0.9896 | 0.9868 |
| 6 | 0.9999 | 0.9999 | 0.9997 | 0.9996 | 0.9994 | 0.9991 | 0.9987 | 0.9981 | 0.9974 | 0.9966 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9994 | 0.9992 |
| 8 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 2.00 | 2.10 | 2.20 | 2.30 | 2.40 | 2.50 | 2.60 | 2.70 | 2.80 | 2.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.1353 | 0.1225 | 0.1108 | 0.1003 | 0.0907 | 0.0821 | 0.0743 | 0.0672 | 0.0608 | 0.0550 |
| 1 | 0.4060 | 0.3796 | 0.3546 | 0.3309 | 0.3084 | 0.2873 | 0.2674 | 0.2487 | 0.2311 | 0.2146 |
| 2 | 0.6767 | 0.6496 | 0.6227 | 0.5960 | 0.5697 | 0.5438 | 0.5184 | 0.4936 | 0.4695 | 0.4460 |
| 3 | 0.8571 | 0.8386 | 0.8194 | 0.7993 | 0.7787 | 0.7576 | 0.7360 | 0.7141 | 0.6919 | 0.6696 |
| 4 | 0.9473 | 0.9379 | 0.9275 | 0.9162 | 0.9041 | 0.8912 | 0.8774 | 0.8629 | 0.8477 | 0.8318 |
| 5 | 0.9834 | 0.9796 | 0.9751 | 0.9700 | 0.9643 | 0.9580 | 0.9510 | 0.9433 | 0.9349 | 0.9258 |
| 6 | 0.9955 | 0.9941 | 0.9925 | 0.9906 | 0.9884 | 0.9858 | 0.9828 | 0.9794 | 0.9756 | 0.9713 |
| 7 | 0.9989 | 0.9985 | 0.9980 | 0.9974 | 0.9967 | 0.9958 | 0.9947 | 0.9934 | 0.9919 | 0.9901 |
| 8 | 0.9998 | 0.9997 | 0.9995 | 0.9994 | 0.9991 | 0.9989 | 0.9985 | 0.9981 | 0.9976 | 0.9969 |
| 9 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9991 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 3.00 | 3.10 | 3.20 | 3.30 | 3.40 | 3.50 | 3.60 | 3.70 | 3.80 | 3.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.0498 | 0.0450 | 0.0408 | 0.0369 | 0.0334 | 0.0302 | 0.0273 | 0.0247 | 0.0224 | 0.0202 |
| 1 | 0.1991 | 0.1847 | 0.1712 | 0.1586 | 0.1468 | 0.1359 | 0.1257 | 0.1162 | 0.1074 | 0.0992 |
| 2 | 0.4232 | 0.4012 | 0.3799 | 0.3594 | 0.3397 | 0.3208 | 0.3027 | 0.2854 | 0.2689 | 0.2531 |
| 3 | 0.6472 | 0.6248 | 0.6025 | 0.5803 | 0.5584 | 0.5366 | 0.5152 | 0.4942 | 0.4735 | 0.4532 |
| 4 | 0.8153 | 0.7982 | 0.7806 | 0.7626 | 0.7442 | 0.7254 | 0.7064 | 0.6872 | 0.6678 | 0.6484 |
| 5 | 0.9161 | 0.9057 | 0.8946 | 0.8829 | 0.8705 | 0.8576 | 0.8441 | 0.8301 | 0.8156 | 0.8006 |
| 6 | 0.9665 | 0.9612 | 0.9554 | 0.9490 | 0.9421 | 0.9347 | 0.9267 | 0.9182 | 0.9091 | 0.8995 |
| 7 | 0.9881 | 0.9858 | 0.9832 | 0.9802 | 0.9769 | 0.9733 | 0.9692 | 0.9648 | 0.9599 | 0.9546 |
| 8 | 0.9962 | 0.9953 | 0.9943 | 0.9931 | 0.9917 | 0.9901 | 0.9883 | 0.9863 | 0.9840 | 0.9815 |
| 9 | 0.9989 | 0.9986 | 0.9982 | 0.9978 | 0.9973 | 0.9967 | 0.9960 | 0.9952 | 0.9942 | 0.9931 |
| 10 | 0.9997 | 0.9996 | 0.9995 | 0.9994 | 0.9992 | 0.9990 | 0.9987 | 0.9984 | 0.9981 | 0.9977 |
| 11 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9994 | 0.9993 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

## CUMULATIVE POISSON PROBABILITIES

| λ | 4.00 | 4.10 | 4.20 | 4.30 | 4.40 | 4.50 | 4.60 | 4.70 | 4.80 | 4.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.0183 | 0.0166 | 0.0150 | 0.0136 | 0.0123 | 0.0111 | 0.0101 | 0.0091 | 0.0082 | 0.0074 |
| 1 | 0.0916 | 0.0845 | 0.0780 | 0.0719 | 0.0663 | 0.0611 | 0.0563 | 0.0518 | 0.0477 | 0.0439 |
| 2 | 0.2381 | 0.2238 | 0.2102 | 0.1974 | 0.1851 | 0.1736 | 0.1626 | 0.1523 | 0.1425 | 0.1333 |
| 3 | 0.4335 | 0.4142 | 0.3954 | 0.3772 | 0.3594 | 0.3423 | 0.3257 | 0.3097 | 0.2942 | 0.2793 |
| 4 | 0.6288 | 0.6093 | 0.5898 | 0.5704 | 0.5512 | 0.5321 | 0.5132 | 0.4946 | 0.4763 | 0.4582 |
| 5 | 0.7851 | 0.7693 | 0.7531 | 0.7367 | 0.7199 | 0.7029 | 0.6858 | 0.6684 | 0.6510 | 0.6335 |
| 6 | 0.8893 | 0.8786 | 0.8675 | 0.8558 | 0.8436 | 0.8311 | 0.8180 | 0.8046 | 0.7908 | 0.7767 |
| 7 | 0.9489 | 0.9427 | 0.9361 | 0.9290 | 0.9214 | 0.9134 | 0.9049 | 0.8960 | 0.8867 | 0.8769 |
| 8 | 0.9786 | 0.9755 | 0.9721 | 0.9683 | 0.9642 | 0.9597 | 0.9549 | 0.9497 | 0.9442 | 0.9382 |
| 9 | 0.9919 | 0.9905 | 0.9889 | 0.9871 | 0.9851 | 0.9829 | 0.9805 | 0.9778 | 0.9749 | 0.9717 |
| 10 | 0.9972 | 0.9966 | 0.9959 | 0.9952 | 0.9943 | 0.9933 | 0.9922 | 0.9910 | 0.9896 | 0.9880 |
| 11 | 0.9991 | 0.9989 | 0.9986 | 0.9983 | 0.9980 | 0.9976 | 0.9971 | 0.9966 | 0.9960 | 0.9953 |
| 12 | 0.9997 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9992 | 0.9990 | 0.9988 | 0.9986 | 0.9983 |
| 13 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9997 | 0.9996 | 0.9995 | 0.9994 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9998 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 16 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| λ | 5.00 | 5.50 | 6.00 | 6.50 | 7.00 | 7.50 | 8.00 | 8.50 | 9.00 | 9.50 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.0067 | 0.0041 | 0.0025 | 0.0015 | 0.0009 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0001 |
| 1 | 0.0404 | 0.0266 | 0.0174 | 0.0113 | 0.0073 | 0.0047 | 0.0030 | 0.0019 | 0.0012 | 0.0008 |
| 2 | 0.1247 | 0.0884 | 0.0620 | 0.0430 | 0.0296 | 0.0203 | 0.0138 | 0.0093 | 0.0062 | 0.0042 |
| 3 | 0.2650 | 0.2017 | 0.1512 | 0.1118 | 0.0818 | 0.0591 | 0.0424 | 0.0301 | 0.0212 | 0.0149 |
| 4 | 0.4405 | 0.3575 | 0.2851 | 0.2237 | 0.1730 | 0.1321 | 0.0996 | 0.0744 | 0.0550 | 0.0403 |
| 5 | 0.6160 | 0.5289 | 0.4457 | 0.3690 | 0.3007 | 0.2414 | 0.1912 | 0.1496 | 0.1157 | 0.0885 |
| 6 | 0.7622 | 0.6860 | 0.6063 | 0.5265 | 0.4497 | 0.3782 | 0.3134 | 0.2562 | 0.2068 | 0.1649 |
| 7 | 0.8666 | 0.8095 | 0.7440 | 0.6728 | 0.5987 | 0.5246 | 0.4530 | 0.3856 | 0.3239 | 0.2687 |
| 8 | 0.9319 | 0.8944 | 0.8472 | 0.7916 | 0.7291 | 0.6620 | 0.5925 | 0.5231 | 0.4557 | 0.3918 |
| 9 | 0.9682 | 0.9462 | 0.9161 | 0.8774 | 0.8305 | 0.7764 | 0.7166 | 0.6530 | 0.5874 | 0.5218 |
| 10 | 0.9863 | 0.9747 | 0.9574 | 0.9332 | 0.9015 | 0.8622 | 0.8159 | 0.7634 | 0.7060 | 0.6453 |
| 11 | 0.9945 | 0.9890 | 0.9799 | 0.9661 | 0.9467 | 0.9208 | 0.8881 | 0.8487 | 0.8030 | 0.7520 |
| 12 | 0.9980 | 0.9955 | 0.9912 | 0.9840 | 0.9730 | 0.9573 | 0.9362 | 0.9091 | 0.8758 | 0.8364 |
| 13 | 0.9993 | 0.9983 | 0.9964 | 0.9929 | 0.9872 | 0.9784 | 0.9658 | 0.9486 | 0.9261 | 0.8981 |
| 14 | 0.9998 | 0.9994 | 0.9986 | 0.9970 | 0.9943 | 0.9897 | 0.9827 | 0.9726 | 0.9585 | 0.9400 |
| 15 | 0.9999 | 0.9998 | 0.9995 | 0.9988 | 0.9976 | 0.9954 | 0.9918 | 0.9862 | 0.9780 | 0.9665 |
| 16 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9990 | 0.9980 | 0.9963 | 0.9934 | 0.9889 | 0.9823 |
| 17 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9984 | 0.9970 | 0.9947 | 0.9911 |
| 18 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9993 | 0.9987 | 0.9976 | 0.9957 |
| 19 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 | 0.9989 | 0.9980 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9991 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 |
| 22 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 24 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

## CUMULATIVE POISSON PROBABILITIES

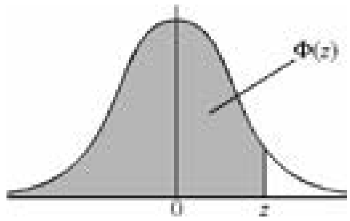
| λ | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 | 15.00 | 16.00 | 17.00 | 18.00 | 19.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x = 0$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0028 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 0.0103 | 0.0049 | 0.0023 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4 | 0.0293 | 0.0151 | 0.0076 | 0.0037 | 0.0018 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 5 | 0.0671 | 0.0375 | 0.0203 | 0.0107 | 0.0055 | 0.0028 | 0.0014 | 0.0007 | 0.0003 | 0.0002 |
| 6 | 0.1301 | 0.0786 | 0.0458 | 0.0259 | 0.0142 | 0.0076 | 0.0040 | 0.0021 | 0.0010 | 0.0005 |
| 7 | 0.2202 | 0.1432 | 0.0895 | 0.0540 | 0.0316 | 0.0180 | 0.0100 | 0.0054 | 0.0029 | 0.0015 |
| 8 | 0.3328 | 0.2320 | 0.1550 | 0.0998 | 0.0621 | 0.0374 | 0.0220 | 0.0126 | 0.0071 | 0.0039 |
| 9 | 0.4579 | 0.3405 | 0.2424 | 0.1658 | 0.1094 | 0.0699 | 0.0433 | 0.0261 | 0.0154 | 0.0089 |
| 10 | 0.5830 | 0.4599 | 0.3472 | 0.2517 | 0.1757 | 0.1185 | 0.0774 | 0.0491 | 0.0304 | 0.0183 |
| 11 | 0.6968 | 0.5793 | 0.4616 | 0.3532 | 0.2600 | 0.1848 | 0.1270 | 0.0847 | 0.0549 | 0.0347 |
| 12 | 0.7916 | 0.6887 | 0.5760 | 0.4631 | 0.3585 | 0.2676 | 0.1931 | 0.1350 | 0.0917 | 0.0606 |
| 13 | 0.8645 | 0.7813 | 0.6815 | 0.5730 | 0.4644 | 0.3632 | 0.2745 | 0.2009 | 0.1426 | 0.0984 |
| 14 | 0.9165 | 0.8540 | 0.7720 | 0.6751 | 0.5704 | 0.4657 | 0.3675 | 0.2808 | 0.2081 | 0.1497 |
| 15 | 0.9513 | 0.9074 | 0.8444 | 0.7636 | 0.6694 | 0.5681 | 0.4667 | 0.3715 | 0.2867 | 0.2148 |
| 16 | 0.9730 | 0.9441 | 0.8987 | 0.8355 | 0.7559 | 0.6641 | 0.5660 | 0.4677 | 0.3751 | 0.2920 |
| 17 | 0.9857 | 0.9678 | 0.9370 | 0.8905 | 0.8272 | 0.7489 | 0.6593 | 0.5640 | 0.4686 | 0.3784 |
| 18 | 0.9928 | 0.9823 | 0.9626 | 0.9302 | 0.8826 | 0.8195 | 0.7423 | 0.6550 | 0.5622 | 0.4695 |
| 19 | 0.9965 | 0.9907 | 0.9787 | 0.9573 | 0.9235 | 0.8752 | 0.8122 | 0.7363 | 0.6509 | 0.5606 |
| 20 | 0.9984 | 0.9953 | 0.9884 | 0.9750 | 0.9521 | 0.9170 | 0.8682 | 0.8055 | 0.7307 | 0.6472 |
| 21 | 0.9993 | 0.9977 | 0.9939 | 0.9859 | 0.9712 | 0.9469 | 0.9108 | 0.8615 | 0.7991 | 0.7255 |
| 22 | 0.9997 | 0.9990 | 0.9970 | 0.9924 | 0.9833 | 0.9673 | 0.9418 | 0.9047 | 0.8551 | 0.7931 |
| 23 | 0.9999 | 0.9995 | 0.9985 | 0.9960 | 0.9907 | 0.9805 | 0.9633 | 0.9367 | 0.8989 | 0.8490 |
| 24 | 1.0000 | 0.9998 | 0.9993 | 0.9980 | 0.9950 | 0.9888 | 0.9777 | 0.9594 | 0.9317 | 0.8933 |
| 25 | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9974 | 0.9938 | 0.9869 | 0.9748 | 0.9554 | 0.9269 |
| 26 | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9987 | 0.9967 | 0.9925 | 0.9848 | 0.9718 | 0.9514 |
| 27 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9983 | 0.9959 | 0.9912 | 0.9827 | 0.9687 |
| 28 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 | 0.9978 | 0.9950 | 0.9897 | 0.9805 |
| 29 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9989 | 0.9973 | 0.9941 | 0.9882 |
| 30 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9986 | 0.9967 | 0.9930 |
| 31 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9993 | 0.9982 | 0.9960 |
| 32 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9990 | 0.9978 |
| 33 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9995 | 0.9988 |
| 34 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 |
| 35 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 |
| 36 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 37 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 38 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

## THE NORMAL DISTRIBUTION FUNCTION

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $z$, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \leqslant z).$$

For negative values of $z$ use $\Phi(-z) = 1 - \Phi(z)$.




| $z$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | ADD | | | | |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 | 4 | 8 | 12 | 15 | 19 | 23 | 27 | 31 | 35 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 | 4 | 7 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 | 3 | 7 | 10 | 14 | 17 | 20 | 24 | 27 | 31 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 | 3 | 7 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 | 3 | 5 | 8 | 11 | 14 | 16 | 19 | 22 | 25 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 | 3 | 5 | 8 | 10 | 13 | 15 | 18 | 20 | 23 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## Critical values for the normal distribution

If $Z$ has a normal distribution with mean 0 and variance 1 then, for each value of $p$, the table gives the value of $z$ such that

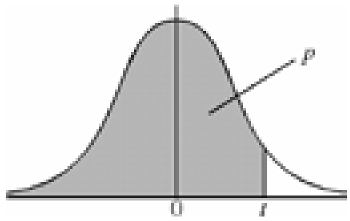
$$\mathrm{P}(Z \leqslant z) = p.$$

| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|
| $z$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

## CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If $T$ has a $t$-distribution with $\nu$ degrees of freedom then, for each pair of values of $p$ and $\nu$, the table gives the value of $t$ such that:

$P(T \leqslant t) = p$.




| $p$ | 0.75 | 0.90 | 0.95 | 0.975 | 0.99 | 0.995 | 0.9975 | 0.999 | 0.9995 |
|---|---|---|---|---|---|---|---|---|---|
| $\nu = 1$ | 1.000 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 4.773 | 5.894 | 6.869 |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.197 | 3.610 | 3.922 |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.057 | 3.421 | 3.689 |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.038 | 3.396 | 3.660 |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 2.860 | 3.160 | 3.373 |
| $\infty$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 |

# Mathematical notation

Examinations for the syllabus in this booklet may use relevant notation from the following list.

**1 Set notation**

| | |
|---|---|
| $\in$ | is an element of |
| $\notin$ | is not an element of |
| $\{x_1, x_2, \ldots\}$ | the set with elements $x_1, x_2, \ldots$ |
| $\{x : \ldots\}$ | the set of all $x$ such that … |
| $\mathrm{n}(A)$ | the number of elements in set $A$ |
| $\varnothing$ | the empty set |
| $\mathscr{E}$ | the universal set |
| $A'$ | the complement of the set $A$ |
| $\mathbb{N}$ | the set of natural numbers, $\{1, 2, 3, \ldots\}$ |
| $\mathbb{Z}$ | the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$ |
| $\mathbb{Z}^+$ | the set of positive integers, $\{1, 2, 3, \ldots\}$ |
| $\mathbb{Z}_n$ | the set of integers modulo $n$, $\{0, 1, 2, \ldots, n-1\}$ |
| $\mathbb{Q}$ | the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$ |
| $\mathbb{Q}^+$ | the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$ |
| $\mathbb{Q}_0^+$ | set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geqslant 0\}$ |
| $\mathbb{R}$ | the set of real numbers |
| $\mathbb{R}^+$ | the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$ |
| $\mathbb{R}_0^+$ | the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geqslant 0\}$ |
| $\mathbb{C}$ | the set of complex numbers |
| $(x, y)$ | the ordered pair $x, y$ |
| $A \times B$ | the cartesian product of sets $A$ and $B$, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$ |
| $\subseteq$ | is a subset of |
| $\subset$ | is a proper subset of |
| $\cup$ | union |
| $\cap$ | intersection |
| $[a, b]$ | the closed interval $\{x \in \mathbb{R} : a \leqslant x \leqslant b\}$ |
| $[a, b)$ | the interval $\{x \in \mathbb{R} : a \leqslant x < b\}$ |
| $(a, b]$ | the interval $\{x \in \mathbb{R} : a < x \leqslant b\}$ |
| $(a, b)$ | the open interval $\{x \in \mathbb{R} : a < x < b\}$ |
| $y\,R\,x$ | $y$ is related to $x$ by the relation $R$ |
| $y \sim x$ | $y$ is equivalent to $x$, in the context of some equivalence relation |

## 2 Miscellaneous symbols

| | |
|---|---|
| $=$ | is equal to |
| $\neq$ | is not equal to |
| $\equiv$ | is identical to or is congruent to |
| $\approx$ | is approximately equal to |
| $\cong$ | is isomorphic to |
| $\propto$ | is proportional to |
| $<$ | is less than |
| $\leqslant$ | is less than or equal to |
| $>$ | is greater than |
| $\geqslant$ | is greater than or equal to |
| $\infty$ | infinity |
| $p \wedge q$ | $p$ and $q$ |
| $p \vee q$ | $p$ or $q$ (or both) |
| $\sim p$ | not $p$ |
| $p \Rightarrow q$ | $p$ implies $q$ (if $p$ then $q$) |
| $p \Leftarrow q$ | $p$ is implied by $q$ (if $q$ then $p$) |
| $p \Leftrightarrow q$ | $p$ implies and is implied by $q$ ($p$ is equivalent to $q$) |
| $\exists$ | there exists |
| $\forall$ | for all |

## 3 Operations

| | |
|---|---|
| $a + b$ | $a$ plus $b$ |
| $a - b$ | $a$ minus $b$ |
| $a \times b,\ ab,\ a.b$ | $a$ multiplied by $b$ |
| $a \div b,\ \frac{a}{b},\ a / b$ | $a$ divided by $b$ |
| $\sum_{i=1}^{n} a_i$ | $a_1 + a_2 + \ldots + a_n$ |
| $\prod_{i=1}^{n} a_i$ | $a_1 \times a_2 \times \ldots \times a_n$ |
| $\sqrt{a}$ | the positive square root of $a$ |
| $\lvert a \rvert$ | the modulus of $a$ |
| $n!$ | $n$ factorial |
| $\binom{n}{r}$ | the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for $n \in \mathbb{Z}^+$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$ |

## 4 Functions

| | |
|---|---|
| $\mathrm{f}(x)$ | the value of the function f at $x$ |
| $\mathrm{f} : A \rightarrow B$ | f is a function under which each element of set $A$ has an image in set $B$ |
| $\mathrm{f} : x \mapsto y$ | the function f maps the element $x$ to the element $y$ |
| $\mathrm{f}^{-1}$ | the inverse function of the function f |
| gf | the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$ |
| $\lim_{x \to a} \mathrm{f}(x)$ | the limit of $\mathrm{f}(x)$ as $x$ tends to $a$ |

| | |
|---|---|
| $\Delta x, \delta x$ | an increment of $x$ |
| $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ | the derivative of $y$ with respect to $x$ |
| $\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$ | the $n$th derivative of $y$ with respect to $x$ |
| $\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$ | the first, second, ..., $n$th derivatives of $\mathrm{f}(x)$ with respect to $x$ |
| $\int y \, \mathrm{d}x$ | the indefinite integral of $y$ with respect to $x$ |
| $\int_a^b y \, \mathrm{d}x$ | the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$ |
| $\dfrac{\partial V}{\partial x}$ | the partial derivative of $V$ with respect to $x$ |
| $\dot{x}, \ddot{x}, \ldots$ | the first, second, ... derivatives of $x$ with respect to $t$ |

## 5 Exponential and logarithmic functions

| | |
|---|---|
| $\mathrm{e}$ | base of natural logarithms |
| $\mathrm{e}^x, \exp x$ | exponential function of $x$ |
| $\log_a x$ | logarithm to the base $a$ of $x$ |
| $\ln x, \log_{\mathrm{e}} x$ | natural logarithm of $x$ |
| $\lg x, \log_{10} x$ | logarithm of $x$ to base $10$ |

## 6 Circular and hyperbolic functions

| | |
|---|---|
| $\left.\begin{array}{l}\sin, \cos, \tan \\ \operatorname{cosec}, \sec, \cot\end{array}\right\}$ | the circular functions |
| $\left.\begin{array}{l}\sin^{-1}, \cos^{-1}, \tan^{-1} \\ \operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1}\end{array}\right\}$ | the inverse circular functions |
| $\left.\begin{array}{l}\sinh, \cosh, \tanh \\ \operatorname{cosech}, \operatorname{sech}, \coth\end{array}\right\}$ | the hyperbolic functions |
| $\left.\begin{array}{l}\sinh^{-1}, \cosh^{-1}, \tanh^{-1} \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \coth^{-1}\end{array}\right\}$ | the inverse hyperbolic functions |

## 7 Complex numbers

| | |
|---|---|
| $\mathrm{i}$ | square root of $-1$ |
| $z$ | a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$ |
| $\operatorname{Re} z$ | the real part of $z$, $\operatorname{Re} z = x$ |
| $\operatorname{Im} z$ | the imaginary part of $z$, $\operatorname{Im} z = y$ |
| $\lvert z \rvert$ | the modulus of $z$, $\lvert z \rvert = \sqrt{x^2 + y^2}$ |
| $\arg z$ | the argument of $z$, $\arg z = \theta, -\pi < \theta \leqslant \pi$ |
| $z^*$ | the complex conjugate of $z$, $x - \mathrm{i}y$ |

## 8 Matrices

| | |
|---|---|
| $\mathbf{M}$ | a matrix $\mathbf{M}$ |
| $\mathbf{M}^{-1}$ | the inverse of the matrix $\mathbf{M}$ |
| $\mathbf{M}^{\mathrm{T}}$ | the transpose of the matrix $\mathbf{M}$ |
| $\det \mathbf{M}$ or $\lvert \mathbf{M} \rvert$ | the determinant of the square matrix $\mathbf{M}$ |

## 9 Vectors

| | |
|---|---|
| $\mathbf{a}$ | the vector $\mathbf{a}$ |
| $\overrightarrow{AB}$ | the vector represented in magnitude and direction by the directed line segment $AB$ |
| $\hat{\mathbf{a}}$ | a unit vector in the direction of $\mathbf{a}$ |
| $\mathbf{i}, \mathbf{j}, \mathbf{k}$ | unit vectors in the directions of the cartesian coordinate axes |
| $\lvert \mathbf{a} \rvert, a$ | the magnitude of $\mathbf{a}$ |
| $\lvert \overrightarrow{AB} \rvert, AB$ | the magnitude of $\overrightarrow{AB}$ |
| $\mathbf{a.b}$ | the scalar product of $\mathbf{a}$ and $\mathbf{b}$ |
| $\mathbf{a} \times \mathbf{b}$ | the vector product of $\mathbf{a}$ and $\mathbf{b}$ |

## 10 Probability and statistics

| | |
|---|---|
| $A, B, C,$ etc. | events |
| $A \cup B$ | union of the events $A$ and $B$ |
| $A \cap B$ | intersection of the events $A$ and $B$ |
| $\mathrm{P}(A)$ | probability of the event $A$ |
| $A'$ | complement of the event $A$ |
| $\mathrm{P}(A \mid B)$ | probability of the event $A$ conditional on the event $B$ |
| $X, Y, R,$ etc. | random variables |
| $x, y, r,$ etc. | values of the random variables $X, Y, R,$ etc. |
| $x_1, x_2, \ldots$ | observations |
| $f_1, f_2, \ldots$ | frequencies with which the observations $x_1, x_2, \ldots$ occur |
| $\mathrm{p}(x)$ | probability function $\mathrm{P}(X = x)$ of the discrete random variable $X$ |
| $p_1, p_2, \ldots$ | probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable $X$ |
| $\mathrm{f}(x), \mathrm{g}(x), \ldots$ | the value of the probability density function of a continuous random variable $X$ |
| $\mathrm{F}(x), \mathrm{G}(x), \ldots$ | the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of a continuous random variable $X$ |
| $\mathrm{E}(X)$ | expectation of the random variable $X$ |
| $\mathrm{E}(\mathrm{g}(X))$ | expectation of $\mathrm{g}(X)$ |
| $\mathrm{Var}(X)$ | variance of the random variable $X$ |
| $\mathrm{G}(t)$ | probability generating function for a random variable which takes the values $0, 1, 2, \ldots$ |
| $\mathrm{B}(n, p)$ | binomial distribution with parameters $n$ and $p$ |
| $\mathrm{Geo}(p)$ | geometric distribution with parameter $p$ |
| $\mathrm{Po}(\lambda)$ | Poisson distribution with parameter $\lambda$ |
| $\mathrm{N}(\mu, \sigma^2)$ | normal distribution with mean $\mu$ and variance $\sigma^2$ |
| $\mu$ | population mean |
| $\sigma^2$ | population variance |
| $\sigma$ | population standard deviation |
| $\bar{x}$, m | sample mean |
| $s^2, \hat{\sigma}^2$ | unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\Sigma(x_i - \bar{x})^2$ |
| $\phi$ | probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$ |
| $\Phi$ | corresponding cumulative distribution function |

# Additional information

## Equality and inclusion

This syllabus complies with our *Code of Practice* and *Ofqual General Conditions of Recognition*.

We have taken great care in the preparation of this syllabus and related assessment materials to avoid bias of any kind. To comply with the UK Equality Act (2010), we have designed this qualification with the aim of avoiding direct and indirect discrimination.

The standard assessment arrangements may present unnecessary barriers for candidates with disabilities or learning difficulties. Arrangements can be put in place for these candidates to enable them to access the assessments and receive recognition of their attainment. Access arrangements will not be agreed if they give candidates an unfair advantage over others or if they compromise the standards being assessed. Candidates who are unable to access the assessment of any component may be eligible to receive an award based on the parts of the assessment they have taken. Information on access arrangements is found in the *Cambridge Handbook (UK)*, for the relevant year, which can be downloaded from the website **www.cambridgeinternational.org/eoguide**

## Guided learning hours

Cambridge Pre-U syllabuses are designed on the assumption that learners have around 380 guided learning hours per Principal Subject over the duration of the course, but this is for guidance only. The number of hours may vary according to curricular practice and the learners' prior experience of the subject.

## Total qualification time

This syllabus has been designed assuming that the total qualification time per subject will include both guided learning and independent learning activities. The estimated number of guided learning hours for this syllabus is 380 hours over the duration of the course. The total qualification time for this syllabus has been estimated to be approximately 500 hours per subject over the duration of the course. These values are guidance only. The number of hours required to gain the qualification may vary according to the local curricular practice and the learners' prior experience of the subject.

## If you are not yet a Cambridge school

Learn about the benefits of becoming a Cambridge school at **www.cambridgeinternational.org/startcambridge**. Email us at **info@cambridgeinternational.org** to find out how your organisation can register to become a Cambridge school.

## Language

This syllabus and the associated assessment materials are available in English only.

Cambridge Assessment International Education
The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA
Tel: +44 (0)1223 553554 Fax: +44 (0)1223 553558
Email: info@cambridgeinternational.org www.cambridgeinternational.org